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Partitioning Graphs into Monochromatic Trees

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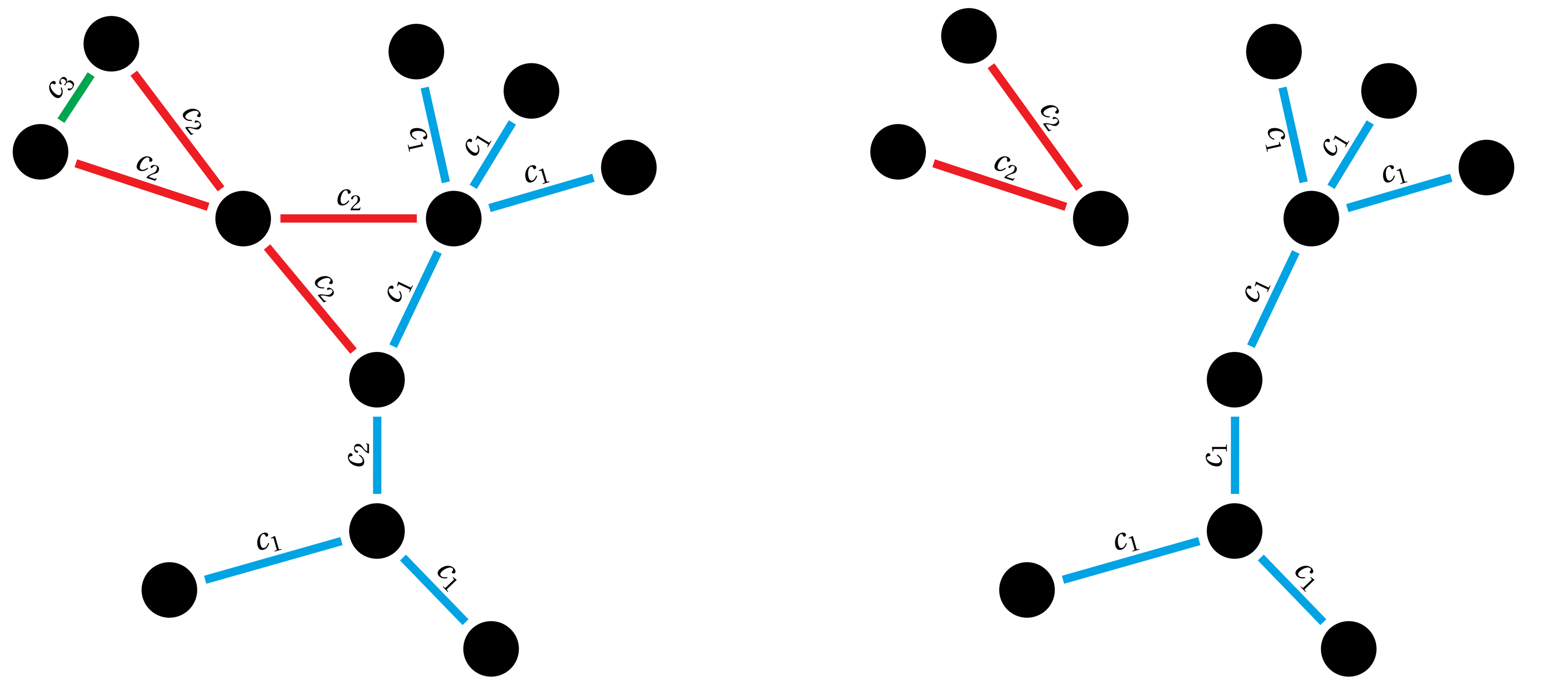
Introduction

This work presents complexity results about the NP-Completeness of Partition edge-coloured Graphs into vertex disjoint Monochromatic Trees (**PGMT**) when we restrict the frequency with each color occurs at the edges of the graph. Jin and Li [3] defined the the PGMT problem as follows:

THE PGMT PROBLEM
Instance: An edge-coloured graph G and a positive integer k.
Question: Are there k or less vertex disjoint monochromatic trees which cover the vertices of the graph G?

Figure 1 shows an example of a graph partitioned into monochromatic trees; even in the colored graph with 3 colors, only two trees are sufficient to cover the vertices.



(a) (b)

Figure 1

Related Works

- In their work, Jin and Li [3] showed that **PGMT** is *NP-Complete* and there is no constant factor approximation algorithm.
- Jin and Li [4], defined a more restricted version of the problem. In this version, the number of distinct colors of G is fixed, and this version is known as **r-PGMT**, where r is the number of colors. For all $r \geq 5$, they showed that **r-PGMT** is also *NP-Complete*.
- Jin et al. [2] showed that, for $r = 2$, **r-PGMT** is also *NP-Complete* for bipartite graphs. For complete bipartite and complete multipartite graphs, however, they presented algorithms that solve the problem in polynomial time.

The f_{MAX}-PGMT Problem

Jin and Li [4] considered a version of **PGMT** where the number of different colors of the graph is fixed. In this work we consider another kind of restriction to the input graph. In this version, instead of fixing the number of different colors, we only guaranteed that each color appears at most f times. We define this version as follows:

THE f_{MAX}-PGMT PROBLEM
Instance: An edge-coloured graph G, where each color occurs at most f times, and a positive integer k.
Question: Are there k or less vertex disjoint monochromatic trees which cover the vertices of the graph G?

NP-Completeness Results

We now show that f_{MAX}-PGMT is *NP-Complete*, when $f = 3$, by reducing from *Exact Cover by 3-Sets* - **X3C** [1], which is defined as follows:

The X3C Problem
Instance: An set $\mathcal{X} = \{v_1, ..., v_n\}$, $|\mathcal{X}| = 3k$; an family of subsets $\mathcal{F} = \{S_1, S_2, ..., S_m\}$, $S_i \subseteq \mathcal{X}$ e $|S_i| = 3$, $i \in \{1, 2, ..., |\mathcal{F}|\}$.
Question: Is there $\mathcal{F}' \subseteq \mathcal{F}$, such that $\bigcup_{S \in \mathcal{F}'} S = \mathcal{X}$?

We build an instance $(G, k + m - 2)$ of f_{MAX}**-PGMT** that is equivalent to an instance $(\mathcal{X}, \mathcal{F})$ of **X3C** as follows: The set of vertices is $V(G) = \{v_1, ..., v_n, S1, ..., S_m, z_1, ..., z_{m-2}\}$. The set of edges is

$$E(G) = \begin{cases} v_i S_j, & if\ v_i \in S_j \\ z_i S_p \end{cases} \tag{1}$$

for all $i \in \{1, ..., n\},\ j \in \{1, ..., m\},\ p \in \{i, i+1, i+2\}$. And coloring the edges as follow:

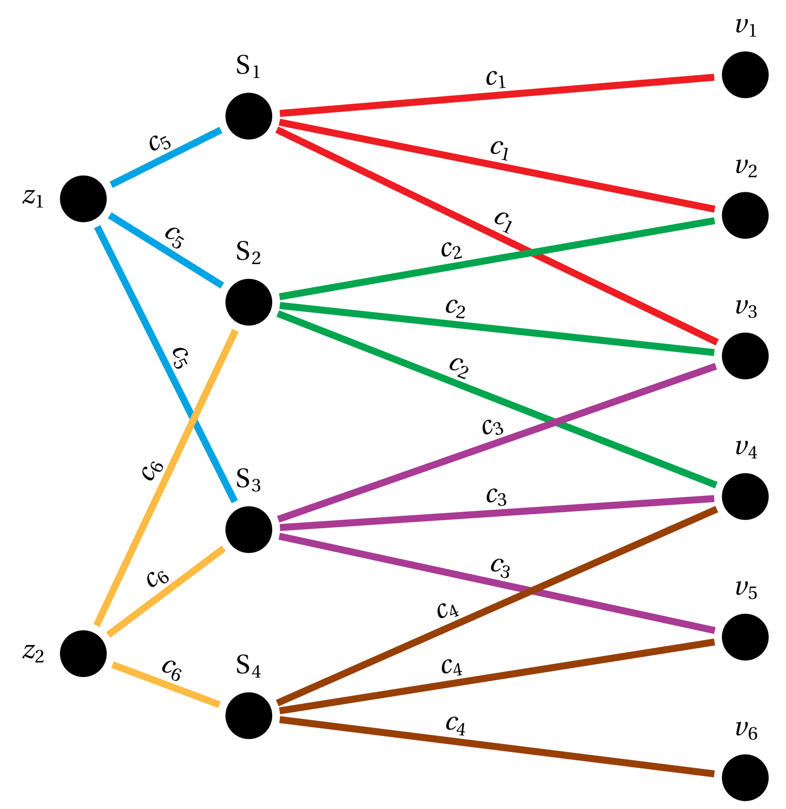
$$c(e) = \begin{cases} c_j & , if\ e = v_i S_j \\ c_{m+p} & , if\ e = z_p S_q \end{cases} \tag{2}$$

for all $e \in E(G),\ i \in \{1, ..., n\},\ j \in \{1, ..., m\},\ p \in \{1, ..., m-2\},\ q \in \{p, p+1, p+2\}$. Figure 2 shows an example of the transformation described: (a) **X3C** instance and (b) colored graph from that instance.

$$\mathcal{X} = \{v_1, v_2, v_3, v_4, v_5, v_6\}$$

$$\mathcal{F} = \begin{Bmatrix} S_1 = \{v_1, v_2, v_3\}, \\ S_2 = \{v_2, v_3, v_4\}, \\ S_3 = \{v_3, v_4, v_5\}, \\ S_4 = \{v_4, v_5, v_6\} \end{Bmatrix}$$

(a)



(b)

Figure 2

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