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11TH LATIN AMERICAN WORKSHOP ON CLIQUES IN GRAPHS

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Preface

The Latin American Workshop on Cliques in Graphs (LAWCG) was originally planned to group the Latin American Graph Theory and Combinatorics community, whose research interests where related to cliques, clique graphs, the behavior of cliques, but nowadays it has presentations in several topics in Graph Theory and related areas.

This 11th edition of this biannual event is being held in Aquiraz, Ceará, Brazil, from October 20th to 23rd, 2024. This is the first time that the workshop is held in the Northeast region of Brazil. This year we also celebrate 25 years of the foundation of the Parallelism, Graphs and Optimization Research Group (ParGO), whose members are responsible for the organization of LAWCG'24.

We would like to thank our invited speakers César Hernández Cruz (UNAM, México), Andrea Jiménez (Universidad de Valparaíso, Chile), Nicolas Nisse (Inria, France), and Marina Esther Groshaus (UTFPR, Brazil) to come and brighten our event with their expertise.

This book contains the abstracts of 53 works presented in the workshop, besides the 4 abstracts of the plenary sessions.

This edition has a novelty. We have a panel discussion on the scientific dissemination in theoretical areas. We hope that this initiative can be a source of inspiration for the next editions to open space to further political discussions among the researchers with similar interests.

Fortaleza, Brazil, October 20th, 2024.

Júlio César Silva Araújo
(General Chair)

LAWCG 2024

WORKSHOP ON CLIQUES IN GRAPHS

Program

20/10/2024 - Sunday	
08:00 - 19:30	
19:30 - 20:30	**Reception Cocktail**

21/10/2024 - Monday		
09:00 - 09:20	Opening Ceremony	
09:20 - 10:10	**Plenary Talk: César Hernández Cruz**	
10:10 - 10:30	Technical Session 1	Technical Session 2
10:30 - 10:50		
10:50 - 11:20	Coffee-break	
11:20 - 11:40	Technical Session 3	Technical Session 4
11:40 - 12:00		
12:00 - 12:20		
12:20 - 12:40		
12:40 - 14:20	Lunch	
14:20 - 14:40	Technical Session 5	Technical Session 6
14:40 - 15:00		
15:00 - 15:20		
15:20 - 15:40		
15:40 - 16:00		
16:00 - 16:30	Coffee-break	
16:30 - 18:00	Panel discussion	

22/10/2024 - Tuesday		
09:00 - 09:50	**Plenary Talk: Andrea Jiménez**	
09:50 - 10:10	Technical Session 7	Technical Session 8
10:10 - 10:30		
10:30 - 10:50		
10:50 - 11:20	Coffee-break	
11:20 - 11:40	Technical Session 9	Technical Session 10
11:40 - 12:00		
12:00 - 12:20		
12:20 - 12:40		
12:40 - 14:40	Lunch	
14:40 - 15:00	Photo of the event	
15:00 - 17:00	Social Activities	
19:00 - 22:00	**Conference Dinner**	

23/10/2024 - Wednesday		
09:00 - 09:50	**Plenary Talk: Nicolas Nisse**	
09:50 - 10:10	Technical Session 11	Technical Session 12
10:10 - 10:30		
10:30 - 10:50		
10:50 - 11:20	Coffee-break	
11:20 - 11:40	Technical Session 13	Technical Session 14
11:40 - 12:00		
12:00 - 12:20		
12:20 - 12:40		
12:40 - 14:20	Lunch	
14:20 - 15:10	**Plenary Talk: Marina Esther Groshaus**	
15:10 - 15:30	Technical Session 15	Technical Session 16
15:30 - 15:50		
15:50 - 16:20	Coffee-break	
16:20 - 16:40	Closing Session	

WORKSHOP ON CLIQUES IN GRAPHS

Technical Sessions

Monday - October 21		
	Session 1 - Aquiraz Room	Session 2 - Iguape Room
10:10 - 10:30	**Locally identifying coloring of split graphs** Márcia Cappelle, Hebert Coelho, Robson Medrado de Oliveira	**Dijkstra Hypergraphs** Leonardo de Almeida Cavadas, Luerbio Faria, André Luiz Pires Guedes, Lilian Markenzon, Lucila Maria de Souza Bento, Jayme Luiz Szwarcfiter
10:30 - 10:50	**O polinômio cromático do grafo $C_3 \square P_n$** Mayara Christina, Mauro Nigro, Diana Sasaki	**Um estudo sobre grafos 2-cordais finos e subclasses** Aline Silva Reis, Vinicius Fernandes dos Santos
	Session 3 - Aquiraz Room	Session 4 - Iguape Room
11:20 - 11:40	**The oriented chromatic number of a wheel and of the disjoint union of a wheel with a complete graph** Erika Morais Martins Coelho, Luerbio Faria, Mateus de Paula Ferreira, Sylvain Gravier, Sulamita Klein, Hebert Coelho da Silva	**Some results on the relationship between modular-related parameters of graphs** Vinicius Fernandes dos Santos, Leandro Freitas de Souza
11:40 - 12:00	**About directed backbone colourings of graphs** Júlio Araújo, Rayane Gomes de Castro, Alexandre Talon	**Variations of chordal and dually chordal graphs characterizable by vertex orderings** Kaio Henrique Masse Vieira, Vinicius Fernandes dos Santos
12:00 - 12:20	**Graceful Coloring of cubic graphs: a focus on Snark families** Simone Dantas, Atílio G. Luiz, Paola T. Pantoja	**IIC-comparability graphs** Marina Groshaus, André Luiz Pires Guedes
12:20 - 12:40	**Total-Neighbor-Distinguishing Index by Sums on Generalized Sierpinski graphs** Simone Dantas, Miguel A. Del Rio Palma, Carlos A. Rodríguez Palma	**When do graph covers preserve the clique dynamics of infinite graphs?** Anna Margarethe Limbach, Martin Winter
	Session 5 - Aquiraz Room	Session 6 - Iguape Room
14:20 - 14:40	**Edge Coloring of the Graph $H_{l,p}$** Athos José de Araújo, Diane Castonguay, André da Cunha Ribeiro	**Inclusion graphs of biclique parts of K_3–free graphs** Edmilson Pereira da Cruz, Marina Groshaus, André Luiz Pires Guedes
14:40 - 15:00	**Preenchendo lacunas na coloração de arestas de grafos split** Diego Amaro Costa, Fernanda Couto, Sulamita Klein	**Erdős sparse halves conjecture for blow-ups of Vega graphs** César Bispo, Walner Mendonça, Guilherme Mota, Roberto Parente
15:00 - 15:20	**On the irregular chromatic index of blow-ups of cycles** Pedro Arraes, Guilherme Mota, Carla Lintzmayer, Maycon Sambinelli	**Estudo sobre 3-atribuição de papéis para produtos direto de grafos** Diane Castonguay, Luiz Felipe Belisário Macedo, Julliano Nascimento
15:20 - 15:40	**Flow decomposition on arc-coloured networks** Jonas Costa, Cláudia Linhares Sales, Ana Karolinna Maia, Cláudio Soares de Carvalho Neto	**Transversals of longest paths on cubic pseudo-Halin graphs** Heloisa Lazari, Yoshiko Wakabayashi
15:40 - 16:00	**Diameter Reduction Via Flipping Arcs** Panna Gehér, Max Kölbl, Lydia Mirabel Mendoza-Cadena, Daniel P. Szabo.	

Tuesday - October 22		
	Session 7 - Aquiraz Room	Session 8 - Iguape Room
09:50 - 10:10	**Sufficient conditions that ensure trees to have game chromatic number 4** Simone Dantas, Celina Figueiredo, Ana Luísa Carvalho Furtado, Miguel Del Rio Palma	**The Path Validity Problem** Hebert Coelho, Leslie Richard Foulds, Gustavo Leal, Humberto José Longo
10:10 - 10:30	**Some results on graph coloring games** Eder Figueiredo, Vinicius Fernandes dos Santos	**On Finding Temporal Cycles and Acyclic Labelings** Davi de Andrade, Allen Ibiapina, Andrea Marino, Ana Shirley Silva

10:30 - 10:50	**Results on the connected greedy coloring game** Thiago Marcilon, Ariane Ribeiro	**Inaproximabilidade do *k*-center balanceado** Lehilton Pedrosa, Hugo Kooki Kasuya Rosado
	Session 9 - Aquiraz Room	Session 10 - Iguape Room
11:20 - 11:40	**Infinite families of two Kochol superpositions of Loupekine snarks are Type 1.** Simone Dantas, Miguel A.D.R. Palma, Giovanna A.B. Penao, Diana Sasaki	**Algoritmos exatos para o problema do Número de Grundy** Davi Gomes Florencio, Wladimir Araujo Tavares
11:40 - 12:00	**On the AVD-total chromatic number of 4-regular circulant graphs** Luerbio Faria, Mauro Nigro, Diana Sasaki	**Scheduling Transmissions in Time-Slotted LoRa Wide Area Networks** Christelle Caillouet, Frédéric Havet, Lucas Picasarri-Arrieta, Teiki Rigaud
12:00 - 12:20	**The AVD-total chromatic number of fullerene nanodiscs** Mariana Cruz, Mauro Nigro, Diana Sasaki	**Graphical Traveling Salesman Problem in some graph classes** Thailsson Clementinio, Rosiane de Freitas Rodrigues
12:20 - 12:40	**Equitable Total Coloring of Grid Graphs** Sheila Almeida, Nicolas Crisostimo Eusebio	**Algorithms for the Maximum Clique Problem with Bit-Level Parallelism** Igor Albuquerque, Wladimir Tavares

Wednesday - October 23

	Session 11 - Aquiraz Room	Session 12 - Iguape Room
09:50 - 10:10	**The Firefighter problem on Loupekine snarks** Luerbio Faria, Sérgio Fusquino, Diego Nicodemos, Vinicius Fernandes dos Santos, Diana Sasaki	**[*k*]-Roman Domination on Triangulated Discs** Atilio Luiz, Lucas de Araújo Torres
10:10 - 10:30	**Graph aspects and algorithms of a Tower of Hanoi-London hybrid game** Jonas Costa, Rosiane de Freitas Rodrigues, Lia Martins	**Double Roman Domination on graphs with maximum degree 3** Atilio Luiz, Francisco Anderson da Silva Vieira
10:30 - 10:50	**The Harmonious Colouring Game** Cláudia Linhares Sales, Nicolas Martins, Nicolas Nisse	**Independent locating-dominating sets in some graph classes** Márcia Cappelle, Erika Morais Martins Coelho, Leslie Richard Foulds, Dayllon Vinícius Xavier Lemos, Humberto José Longo
	Session 13 - Aquiraz Room	Session 14 - Iguape Room
11:20 - 11:40	**Jogos partizan de convexidade em grafos** Samuel Araújo, João Marcos Brito, Raquel Folz, Rosiane de Freitas, Rudini Sampaio	**Almost All Complementary Prisms Have Many Pairwise-disjoint Perfect Matchings** Leandro Zatesko
11:40 - 12:00	**Convexity Games on Oriented Graphs** Samuel N. de Araújo, João Marcos Brito, Rudini Sampaio	**Extending a perfect matching to a hamiltonian cycle in some classes of graphs** C. N. Campos, Alessandra Aparecida Pereira
12:00 - 12:20	**Results on *f*-reversible processes** Pedro Fernandes, Thiago Marcilon, Murillo Silva	**The cost of perfection for matchings in Prism graphs** Ingrid Borchert, Diego Nicodemos, Diana Sasaki
12:20 - 12:40	***W*-Hierarchy of Geodetic number problem** Simone Dantas, Vitor Dos Santos Ponciano, Rômulo Luiz Oliveira da Silva, Alessandra B. Verissimo	**On the ℓ-Nested Cut problem: a generalization of Matching Cut** Guilherme Castro Mendes Gomes, Gabriel Lucas Costa Martins
	Session 15 - Aquiraz Room	Session 16 - Iguape Room
15:10 - 15:30	**Parameterized algorithms for thinness via the cluster module number** Flavia Bonomo, Eric Brandwein, Ignasi Sau	**On open-independent dominating sets for some subclasses of cubic graphs** Erika Morais Martins Coelho, Lauane Moraes
15:30 - 15:50	**Chain Traveling Salesmen Problem** Lehilton Pedrosa, Lucas de Oliveira Silva	**Applying Spectral Graph Theory to Coupled Oscillation problems** Felipe Costa Melo Cunha, Ivan Guilhon, Samuel Madeiro, Antênio José da Costa Sampaio, Rudini Sampaio

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Invited Talks

Revisiting full homomorphisms

César Hernández Cruz
Universidad Nacional Autónoma de México, Mexico

For a pair of graphs G and H, a *homomorphism* from G to H is a function $f : V_G \to V_H$ that preserves adjacencies, i.e., for any pair of vertices u and v of G, the existence of the edge uv in G implies the existence of the edge $f(u)f(v)$ in H. A homomorphism is *full* if it also preserves non-adjacencies (the converse implication in the definition of homomorphism also holds). When H is a fixed graph, the problem of determining, for an input graph G, the existence of a full homomorphism to H, is often called the full H-homomorphism problem or the full H-colouring problem. From the complexity point of view, the full H-colouring problem has been solved: For any fixed graph H, the full H-colouring problem is polynomial-time solvable. This classification has deterred further study on full homomorphisms, so little is known in terms of the fine-grained complexity of this problem. In this talk, we will discuss the full H-colouring problem when H belongs to some simple families of graphs, showing that really nice complexities can be obtained in some cases.

Graphs with Women

Andrea Jiménez
Universidad de Valparaíso, Chile

The main purpose of this talk is to expose part of my academic life as a graph theorist through the work made in collaboration with great female colleagues. I plan to tell you who these women are, in which problems we have been working on, and roughly, what we have done together.

On some positional games in graphs

Nicolas Nisse
Inria d'université Côte d'Azur, France

Maker-Breaker game is a classical 2-Player game where two players, Alice and Bob (starting with Alice), alternately select vertices of an hypergraph. Alice wins if she eventually manages to select all vertices of some hyper-edge. Bob wins otherwise, I.e., if he manages to select all vertices of a transversal of the hyper-edges. The problem of deciding the outcome of the game (who is the winner) is known to be PSPACE complete [Schaefer 1978] even if the hyper-edges have size 6 [Rahman,Watson 2021] and to be polynomial if hyper-edges have size 3 [Galliot,Gravier,Sivignon 24+]. To better understand this game, particular hypergraph classes (defined from graphs) have been studied. For instance, given a graph, we may consider the hypergraph with same vertex set and hyper-edges are the closed neighbourhoods of each vertex (Maker-Breaker domination game) [Duchêne, Gledel, Parreau,Renault 2020]. In this talk, we aim at giving a (far to be exhaustive) overview of these games (and their numerous variants). As examples, we will focus on the Largest Connected Subgraph game (where, given a graph G, Alice aims at selecting the vertices of G inducing a connected subgraph as large as possible) and on the H-game (where players select edges instead of vertices and, given a graph G, Alice aims at selecting edges of G inducing a copy of a fixed graph H). We will describe some complexity results in various graph classes and we will mainly focus on open problems.

Biclique graph on a therapist couch: Analyzing their problems

Marina Esther Groshaus
Universidade Tecnológica Federal do Paraná, Brazil

A biclique in a graph is a maximal complete bipartite induced subgraph. The biclique intersection graph was defined over 20 years ago as the biclique graph, $KB(G)$. Although the class of biclique graphs has a "Kraus Type" characterization, it does not lead to an efficient algorithm for the recognition problem. Furthermore, although we know that the problem is in NP, the computational complexity of the biclique graph recognition problem remains unresolved. Since then, we have focused on better understanding this class, obtaining several partial results that provide different strategies to follow. However, we know that this is a bit complicated class to handle. Therefore, in this presentation, we will sit the biclique graph on the therapist's couch and share their deeper issues, many of which are still unsolved. We will understand their true essence and be surprised to discover that, despite their difficulty, complexity, and particularity, they are not alone, on the contrary, they are very well related!

Session 1

Locally identifying coloring of split graphs

R. M. Oliveira* M. R. Cappelle H. Coelho
Instituto de Informática, UFG, Goiânia, Brazil

Keywords: Lid-coloring, split graphs, split-comparability.

A locally identifying coloring (or lid-coloring for short) in a graph is a proper vertex coloring such that, for any edge uv, if u and v have distinct closed neighborhoods, then the set of colors used on vertices of the closed neighborhoods of u and v are distinct. The lid-chromatic number of a graph G, denoted by $\chi_{lid}(G)$, is the minimum number of colors needed in any lid-coloring of G. Esperet et al. [2012] showed that every bipartite graph has lid-chromatic number at most 4. However, they proved that deciding whether a bipartite graph is 3-lid-colorable is an NP-complete problem, where it can be decided in linear time whether a tree is 3-lid-colorable.

The split graph $G = (K \cup S, E)$ is a graph whose set of vertices can be partitioned into a clique K and an independent set S. We will consider partitions $K \cup S$ with K of maximum size. The split graph is a chordal graph where the maximum clique size and its chromatic number are equal to $|K|$. In general, $\chi_{lid}(G)$ is not bounded by a function of the usual chromatic number $\chi(G)$. Esperet et al. [2012] conjectured that every chordal graph G has $\chi_{lid} \leq 2\chi(G)$. They also proved that if G is a split graph, then $\chi_{lid}(G) \leq 2|K| - 1$.

We study the lid-chromatic number on two subclasses of split graphs: those that are a corona products between a complete graph of order m and the complement of a complete graph of order n, and the split-comparability graphs. For theses graphs we present some closed formulas and provide lower and upper bounds.

References

Louis Esperet, Sylvain Gravier, Mickaël Montassier, Pascal Ochem, and Aline Parreau. Locally identifying coloring of graphs. *Electron. J. Comb.*, 19, 2012.

O polinômio cromático do grafo $C_3 \square P_n$

Mayara Christina [1,*] Mauro Nigro[1] Diana Sasaki[1]
[1] Universidade do Estado do Rio de Janeiro

Keywords: coloração de vértices, polinômio cromático, produto cartesiano

Dado um inteiro positivo λ, uma função $f : V \to \{1, 2, \cdots, \lambda\}$ é chamada de coloração própria de λ de um grafo $G = (V, E)$ se $f(u) \neq f(v)$ sempre que os vértices u e v são adjacentes. Dizemos que duas colorações próprias de λ, f e g, de G são distintas se $f(v) \neq g(v)$ para algum vértice v em G. O polinômio cromático de um grafo, denotado por $\chi(G : \lambda)$, é uma função polinomial que expressa o número de colorações próprias distintas para o grafo G utilizando λ cores.

O produto cartesiano entre dois grafos simples G e H, é o grafo $G \square H$, cujo o conjunto de vértices é $V(G) \times V(H)$ e cujo o conjunto de arestas é o conjunto de todos os pares $(u_1, v_1)(u_2, v_2)$ tal que ou $u_1, u_2 \in E(G)$ e $v_1 = v_2$, ou $v_1 v_2 \in E(G)$ e $u_1 = u_2$.

O polinômio cromático de um grafo G pode ser encontrado por meio do método de remoção e contração. O polinômio cromático de vários grafos já foram determinados utilizando esse metódo, como o do grafo $C_3 \square P_n$. Em 2022, Dong e Koh [1] utilizaram um método matricial para determinar polinômios cromáticos e sugeriram sua aplicação para encontrar o polinômio cromático de um grafo definido como o produto cartesiano entre dois grafos caminhos. No ano de 2024, Yadav et al. [2] determinaram os polinômios cromáticos de alguns grafos $P_m \square P_n$, conforme sugerido por Dong e Koh [1]. Neste estudo, seguimos a técnica proposta por Dong e Koh [1] para determinar os polinômios cromáticos dos grafos $C_3 \square P_n$, em consonância com o trabalho prévio de Yadav et al. [2].

Referencias:

[1] F. Dong and K. M. Koh, Foundations of the chromatic polinomyal, in Handbook of the Tutte polynomial and related topics, J. A. Ellis-Monaghan and I. Moffatt, eds., CRC Press, Florida, USA, 2022, ch. 11, pp. 213–251.

[2] R. Yadav, A. Sehgal, S. Sehgal, and A. Malik, The chromatic polynomial of grid graph $P_3 \square P_n$, Journal of Applied Mathematics and Computing,(2024).

Session 2

Dijkstra Hypergraphs

L. A. Cavadas[1,*], L. Faria[1], L. M. S. Bento[1], J. L. Szwarcfiter[2], L. Markenzon[2], A. L. P. Guedes[3]
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[3] Federal University of Paraná, Brazil

Keywords: Reducible Flow Hypergraphs, Parallel and structured programming

Dijkstra Hypergraphs are directed hypergraphs [2] that model the execution of parallel programs through flow hypergraphs. In this work we introduce linear-time algorithms for recognizing elements within this family and for checking isomorphism between them. This class is derived from the concepts of structured programming, multiprogramming and Dijkstra graphs [1].

The recognition process of these hypergraphs involves constructing a bottom-up contractile sequence until a trivial form is reached performed by an algorithm with time complexity of $O(n)$.

For the isomorphism verification between Dijkstra hypergraphs, we propose an algorithm based on generating codes from special subhypergraphs (called prime) of each hypergraph, following a similar approach to the recognition. With time complexity also at $O(n)$, we demonstrate that two Dijkstra Hypergraphs G_1 and G_2 are isomorphic if, and only if, $C(G_1) = C(G_2)$. Here, $C(G_1)$ and $C(G_2)$ represent these unique codes generated from the structural properties of G_1 and G_2, respectively.

References

[1] Bento, Lucila M. S. and Boccardo, Davidson R. and Machado, Raphael C. S. and Miyazawa, Flávio K. and de Sá, Vinícius G. P. and Szwarcfiter, Jayme L. (2019) *Dijkstra graphs*, Elsevier.

[2] Guedes, A. L. P., Markenzon, L., and Faria, L. (2011). *Flow hypergraph reducibility*, Discrete applied mathematics, 159(16):1775–1785.

Um estudo sobre grafos 2-cordais finos e subclasses

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Universidade Federal de Minas Gerais

Keywords: k-ordering, k-coloring, chordal thinness, graph classes

Seja G um grafo com n vértices. Uma ordenação σ é uma função bijetora que mapeia os vértices de G em índices $\{1, 2, \ldots, n\}$, onde tais índices são as posições de cada vértice em σ. Uma k-coloração é uma função que mapeia os vértices de G em k cores, onde cada cor é associada a um valor no conjunto $\{1, 2, \ldots, k\}$. Em uma ordenação $\sigma = \{v_1, \ldots, v_n\}$, um vértice v_j é vizinho à direita de v_i, se $v_j \in N(v_i)$ e $j > i$. Dizemos que um grafo G é *k-cordal fino* (k-CT) se possuir uma ordenação σ e uma k-coloração nas quais para todo vértice v e toda cor c, os vizinhos de v que possuem a cor c, à direita de v na ordenação σ, formam uma clique [1]. Neste caso, dizemos que a ordenação é uma k-ordenação σ e que a k-coloração é uma k-coloração cordal. A *finura cordal* de um grafo G é o menor k tal que G é k-cordal fino. Definimos a classe k-CT como sendo o conjunto de grafos com finura cordal no máximo k. Em particular, um grafo é 1-CT se e somente se ele é cordal.

A estrutura dos grafos k-CT, para $k \geq 2$ ainda é pouco conhecida. Neste trabalho, nos baseamos na metodologia de estudo de inclusão de classes para melhor conhecimento da classe de grafos 2-CT, para compreender sua estrutura. Em particular, investigamos a interseção de 2-CT com grafos bipartidos e a interseção de 2-CT com 2-degenerados, além de mostrar que todo grafo cactus é 2-CT.

Referências

[1] B. Amorim and V. F. dos Santos, "Finura em grafos cordais," in *Anais do VII Encontro de Teoria da Computação*, (Porto Alegre, RS, Brasil), pp. 73–76, SBC, 2022.

Session 3

The oriented chromatic number of a wheel and of the disjoint union of a wheel with a complete graph

E. Coelho[1], H. Coelho[1], L. Faria[2], M. Ferreira[1,*], S. Gravier[4], S. Klein[3]

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[2] Universidade do Estado do Rio de Janeiro

[3] Universidade Federal do Rio de Janeiro

[4] Université Grenoble Alpes / CNRS

Keywords: Oriented Chromatic Number, Disconnected Graphs, Wheel Graphs and Disjoint Union of Graphs.

Let $\overrightarrow{G} = (V, E)$ be an oriented graph, G the underlying graph of $\overrightarrow{G}$ and k be a positive integer. An *oriented k-coloring* of $\overrightarrow{G}$ is a partition of V into k subsets such that there are no two adjacent vertices belonging to the same subset, and all the arcs between a pair of subsets have the same orientation. The *oriented chromatic number* $\chi_o(\overrightarrow{G})$ of $\overrightarrow{G}$ is the smallest k, such that $\overrightarrow{G}$ admits an oriented k-coloring. The *oriented chromatic number* of G, denoted by $\chi_o(G)$, is the maximum of $\chi_o(\overrightarrow{G})$ for all orientations $\overrightarrow{G}$ of G. Given two graphs G and H we say that $G \cup H$ is the *disjoint union* graph of G and H if $V(G \cup H) = V(G) \cup V(H)$ and $E(G \cup H) = E(G) \cup E(H)$. A *wheel* graph W_n has $V(W_n) = \{v_1, v_2, \ldots, v_n, c\}$ and $E(W_n) = \{v_i v_{i+1} : i \in \{1, 2, \ldots, n-1\}\} \cup \{v_n v_1\} \cup \{v_i c : i \in \{1, 2, \ldots, n\}\}$. Wheel graphs consist of a important class having many theoretical and algorithmic applications with an ample literature on coloring problems. In this paper we determine the value of $\chi_o(W_n)$ as $n+1$ when $3 \leq n \leq 6$, 7 whether $n = 7$ and 8 whether $n \geq 8$, producing a polynomial-time algorithm to color any wheel graph. In (Coelho et al. (2024)) we introduced the study of the oriented chromatic number of the disjoint union of graphs, in this work we show that $\chi_o(K_p \cup W_n)$ has an intricate and non-trivial solution, that uses extremal mathematics results on its proof, which for small value of $n \leq 8$ we give exact values and for large values of $n \geq 9$ is $p+2$ or $p+3$.

About directed backbone colourings of graphs

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Keywords: Graph Colouring, Matching, Planar Graph, Backbone Colouring

Let $G = (V, E)$ be a graph and $H = (V, E(H))$ be a subgraph of G, called a *backbone*. We denote by $\chi(G)$ the chromatic number of G. A k-colouring f is a *q-backbone k-colouring* for (G, H) if f is a proper k-colouring of G and $|f(u)-f(v)| \geq q$ for all edges $uv \in E(H)$. The *q-backbone chromatic number* of (G, H), denoted by $\mathrm{BBC}_q(G, H)$, is the smallest integer k for which there exists a q-backbone k-colouring of (G, H) [1].

Circular and list versions of backbone colourings can be found in the literature. We define and study the directed variant of the problem. If $\overrightarrow{H}$ is an acyclic orientation of $H \subseteq G$ then f is a directed q-backbone colouring when it is a proper colouring of G and for each arc $(u, v) \in \overrightarrow{H}$, $f(v)-f(u) \geq q$. Among other results, we prove that:

Theorem 1. *Let G be a graph, $\overrightarrow{H}$ be an orientation of a subgraph of G, and $q \geq 2$ be an integer. Then $BBC_q(G, \overrightarrow{H}) \leq \chi(G)(\operatorname{diam}(\overrightarrow{H}) + 1) + (q - 2)\operatorname{diam}(\overrightarrow{H})$. This bound is tight.*

Due to the Four Colour Theorem, $BBC_2(G, \overrightarrow{M}) \leq 8$, if G is planar and $\overrightarrow{M}$ is an oriented matching of G. If in addition G is triangle-free then Grötzsch's theorem tells us that it can be properly coloured using only 3 colours. Our result shows that $BBC_2(G, \overrightarrow{M}) \leq 6$ for such a graph. We provide an intermediary result between these two cases.

Theorem 2. *If G is a planar graph without C_4 nor C_5 as subgraph, and $\overrightarrow{M}$ is an oriented matching of G, then $\mathrm{BBC}_2(G, \overrightarrow{M}) \leq 7$.*

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Graceful Coloring of cubic graphs: a focus on Snark families ‡

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Keywords: graceful coloring, graph coloring, vertex coloring.

In graph theory, many variations of graph coloring have been studied over time. *Proper vertex coloring* involves assigning colors to the vertices of a graph G such that adjacent vertices have different colors, while *proper edge coloring* assigns colors to the edges of G, ensuring that adjacent edges have different colors. These concepts inspired the creation of graph labelings, which, in a certain way, extended the idea of simultaneously coloring vertices and edges. A *Graceful labeling* is an injective vertex labeling $g : V(G) \to \{0, 1, ..., |E(G)|\}$, that assigns the label $|g(u) - g(v)|$ to each edge uv of G, resulting in pairwise distinct edge labels.

This work focuses on a variant of graceful labeling known as *graceful coloring*, introduced by Gary Chartrand (2015), and further investigated by Bi et al. (2017). A *graceful k-coloring* of G is a proper vertex coloring $f : V(G) \to \{1, 2, ..., k\}$ that induces a proper edge coloring $f' : E(G) \to \{1, 2, ..., k-1\}$ defined by $f'(uv) = |f(u) - f(v)|$. The *graceful chromatic number* $\chi_g(G)$ of G is the minimum k for which a graceful k-coloring of G exists.

In this work, we study the graceful chromatic number for cubic graphs, and determine that the graceful chromatic numbers of Flower snarks and Goldberg snarks are 7 for the first member of each family, and 6 for the remaining members. For the Generalized Blanusa snarks, the graceful chromatic number is 6 for every member of the first family. For the second family it is 6 for the second member and from the fifth member; otherwise is 7.

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Total-Neighbor-Distinguishing Index by Sums on Generalized Sierpiński graphs ‡

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Keywords: TNDI, AVDTC, total coloring.

We consider a proper coloring c of edges and vertices in a simple graph such that the color of a vertex v, denoted by $\mathrm{Sum}(v)$, is the sum of the colors of all the edges incident to v.

The Total-Neighbor-Distinguishing Index by Sums ($\mathrm{TNDI}_{\sum}$) is a total coloring in which $\mathrm{Sum}(v) \neq \mathrm{Sum}(w)$ for every adjacent vertices v and w. Pilsniak and Wozniak (2015) introduced this coloring and established the conjecture that the minimum number of colors needed for a graph G with maximum degree $\Delta(G)$ to have a proper total coloring that distinguishes adjacent vertices by sums is less than or equal to $\Delta(G)+3$. In 2011, Gravier, Kovse and Parreau introduced the concept of generalized Sierpiński graphs, which extends the notion of Sierpiński graphs, by replacing the complete graph in their definition with any graph G.

In this paper, we continue the work of Pilsniak and Wozniak verifying that generalized Sierpiński graphs, when G is a cycle, a bipartite or a complete graph, satisfy the $\mathrm{TNDI}_{\sum}$ Conjecture. Furthermore, we determine the chromatic number by sums for generalized Sierpiński graphs when G is complete bipartite or a regular bipartite graph. Remarkably, these colorings also satisfy the AVDTC Conjecture. We also show that $\mathrm{TNDI}_{\sum}$ Conjecture is satisfied for the graph classes of the regularizations $+S_p^n$ and $++S_p^n$ of Sierpiński graphs S_p^n.

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Session 4

Some results on the relationship between modular-related parameters of graphs

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Keywords: cographs, graph classes, modular decomposition

Given a graph G, the set $M \subseteq V(G)$ is a module if all vertices in M have the same neighborhood outside of M. It is possible to find a decomposition of G into modules, by using a inclusion tree, to obtain useful data on G.

A module M is prime if both $G[M]$ and its complement are connected. The modular width $mw(G)$ of a graph G is maximum number of children of a prime module in its modular decomposition (or 0, if no such module exists). The neighborhood diversity of G, $nd(G)$, is the number of equivalence classes in the relation where u and v are equivalent iff $N(u) - \{v\} = N(v) - \{u\}$. The iterated type partition [2] generalizes this concept: starting with a graph G we iteratively identify vertices in the same equivalence class into a unique new vertex until all equivalence classes are singletons. The iterated type partition number $itp(G)$ corresponds to the number of vertices in the resulting graph. The $\mathcal{G}$-modular cardinality of G, $\mathcal{G}$-$mc(G)$ [1] is the smallest number of modules that G can be partition into, such that the graph induced by each module belongs to the graph class $\mathcal{G}$.

In this work, we characterize when some of those parameters are equal and show that $itp(G)$=cograph-$mc(G)$ holds for every graph.

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Variations of chordal and dually chordal graphs characterizable by vertex orderings

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Keywords: chordal graphs, dually chordal graphs, vertex orderings

Many graph classes have charaterizations with respect to the existence of a *vertex ordering* satisfying certain properties. Two notable examples are *chordal graphs* which can be characterized [3] as the graphs G that admit an ordering of its vertices $v_1, \ldots, v_n$ such that for every v_i, its neighbors $v_j \in N_{G_i}[v_i]$ satisfy $N_{G_i}[v_i] \subseteq N_{G_i}[v_j]$ where G_i is the subgraph of G induced by $\{v_i, \ldots, v_n\}$, and *dually chordal graphs*, which can be characterized [1, 2] as the graphs G that admit an ordering of its vertices $v_1, \ldots, v_n$ such that every v_i has a neighbor $v_j \in N_{G_i}[v_i]$ such that $N^2_{G_i}[v_i] \subseteq N_{G_i}[v_j]$.

By comparing the definition of the orderings that characterize chordal and dually chordal graphs, it is natural to ask which other graph classes arise for different variations. In this work we start a systematic exploration of classes that can be characterized by ordering of similar flavor to the previous ones.

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IIC-comparability graphs

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Keywords: Biclique graph; Comparability graph; IIC; Partially ordered set

Given a poset $\mathcal{P} = (C, \leq)$ and $x \in C$, let $I^-_{\mathcal{P}}(x) = \{y \in C \mid y \leq x\}$ and $I^+_{\mathcal{P}}(x) = \{y \in C \mid x \leq y\}$ be, respectively, the *predecessors* and *successors intervals* of x in $\mathcal{P}$. $\mathcal{P}$ is *interval intersection closed (IIC)* if the sets $\mathcal{I}^-_{\mathcal{P}} = \{I^-_{\mathcal{P}}(x) \mid x \in C\}$ and $\mathcal{I}^+_{\mathcal{P}} = \{I^+_{\mathcal{P}}(x) \mid x \in C\}$ are *closed under intersection*.

The IIC-comparability graphs are the comparability graphs for which exist an IIC poset as a comparability model. We defined this class while studying the recognition problem of biclique graphs. Indeed, the biclique graphs of bipartite graphs are exactly the square graphs of IIC-comparability graphs. Then, the study of the class of IIC-comparability graphs can give relevant information about biclique graphs. Recall that the IIC-comparability graphs recognition problem complexity is still open.

Observe that the IIC-comparability is not an hereditary class, as the 4-*wheel* graph, W_4 (C_4 and an universal vertex), is an IIC-comparability graph while the C_4 is not.

In this work, we generalize this idea, that is, we give necessary conditions for a IIC-comparability graph admitting certain induced subgraphs. Given two graphs G and H, let $G + H$ denote the join of G and H. Let $X = (n_1, n_2, \dots, n_k)$ such that $k \geq 2$ and $n_i \geq 2$, for $1 \leq i \leq k$. Let K_X be the complete k-partite graph $K_{n_1,n_2,\dots,n_k}$ with parts of sizes $n_1, \cdots, n_k$, and let $S_X = K_X + K_{k-1}$.

We prove that in an IIC-comparability graph, every induced subgraph K_X is a subgraph of some induced S_X such that for every vertex $w \notin V(S_X)$ adjacent to some vertex of K_{k-1}, it follows that $N(w) \cap V(K_X)$ is a transversal of the cliques of K_X, that is, it contains at least one of the parts of K_X.

Remark that this necessary condition is not proved to be sufficient for characterizing IIC-comparability graphs.

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When do graph covers preserve the clique dynamics of infinite graphs?

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Keywords: clique dynamics, triangular cover, infinite graphs, local minimal degree, local girth

Given a potentially infinite (but locally finite) graph G, its clique graph kG has as its vertices the cliques of G (i.e. the maximal complete subgraphs), two of which are adjacent in kG if they have non-empty intersection in G. A graph G is *clique divergent* if all iterated clique graphs $kG, k(kG), k^3G, ...$ are pairwise non-isomorphic, or *clique convergent* otherwise.

It turns out that the study of this so-called *clique dynamics* for infinite graphs is qualitatively different from the one for finite graphs. For example, clique convergence and clique divergence of finite graphs is *cover stable*, i.e. preserved under (triangular) graph covers. The classic proof by Larrión and Neumann-Lara is however based on a pigeon hole argument that does not transfer to the infinite case. In previous work we established that a clique convergent graph, finite or infinite, has a clique convergent universal cover. Beyond this, very little is known about the interactions between clique dynamics and the graph cover operation.

We present an instructive counterexample that shows that clique convergence is in fact not necessarily preserved by graph covers on infinite graphs. We then focus on *local conditions* (i.e. conditions on the neighborhoods of vertices) and show that the following are sufficient to imply cover stability: local girth ≥ 7 and local minimum degree ≥ 2; being locally cyclic and of minimum degree ≥ 6. Despite being semantically similar, the respective arguments turn out very different.

This also raises the following question: is the clique dynamics of graphs of local girth ≥ 6 and local minimum degree ≥ 2 cover stable? We discuss this question briefly.

Session 5

Edge Coloring of the Graph $H_{l,p}$

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Keywords: Cayley Graphs, Graphs $H_{l,p}$, Edge Coloring

The family of graphs $H_{l,p}$ is known for its intriguing structure in graph theory, initially conceived in the context of edge partitions and later identified as Cayley Hamiltonian graphs. In this work, we classify the graphs into two distinct categories based on the parity of the parameter p, each exhibiting unique coloring behaviors.

In the first class, when p is even, we observe that the chromatic index of graphs $H_{l,p}$ is $l(l-1)$. This observation stems from the nature of the cycles generated when the elements of the generating set are repeatedly applied to a vertex. Each cycle formed is bipartite, allowing for a coloring with $l(l-1)$ distinct colors. This phenomenon reveals an intrinsic structure of graphs $H_{l,p}$ when p is even, resulting in a relatively simple and predictable coloring property.

However, in the second class, when p is odd, the situation becomes slightly more complex. Here, the chromatic index of graphs $H_{l,p}$ increases to $l(l-1)+1$. The justification for this difference lies in the analysis of the minimum number of colors required to color these graphs, taking into account the parity of p. Adapting the lower bound for the chromatic number allows us to understand the need for an additional color when p is odd, resulting in a slightly more complex coloring compared to the case when p is even.

Therefore, the parity of p emerges as a crucial factor in determining the chromatic index of graphs $H_{l,p}$, delineating a fundamental distinction between the classes. This discovery not only enriches our theoretical understanding of these graphs but also motivates further investigations into their properties and applications in graph theory and related fields.

Preenchendo lacunas na coloração de arestas de grafos split

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Palavras-chave: coloração de arestas, grafos split, grafos split 3-admissíveis

Um grafo G é k-aresta colorível se existe uma atribuição de k cores às arestas de G de modo que arestas incidentes em um mesmo vértice recebam cores distintas. O menor valor de k tal que G é k-aresta colorível é chamado de *índice cromático* e é denotado por $\chi'(G)$. O resultado mais importante acerca de coloração de arestas (Vizing, 1964) afirma que $\Delta \leq \chi'(G) \leq \Delta+1$, para todo grafo G, classificando os grafos em *Classe 1*, se $\chi'(G) = \Delta$ ou *Classe 2*, caso contrário. Surpreendemente, mesmo com apenas duas opções, determinar $\chi'(G)$ é um problema NP-completo (Holyer, 1981). Um grafo G é dito *split* quando $V(G)$ é particionável em uma clique X e um conjunto independente Y. Classificar um grafo split de acordo com χ' é um problema em aberto, solucionado apenas para algumas subclasses desta classe: grafos split com Δ ímpar (Chen, Fu and Ko, 1995), grafos split que possuem pelo menos um vértice v em Y tal que $\left\lceil \frac{|X|}{2} \right\rceil \leq d(v) \leq \frac{\Delta}{2}$ (Almeida et. al, 2012). G é t-admissível se admite uma árvore geradora na qual a maior distância entre vértices adjacentes de G é t (Cai e Corneil, 1995). O menor valor de t tal que G é t-admissível é $\sigma(G)$. Grafos split são 3-admissíveis (Panda e Das, 2010), o que particiona a classe em: split com $\sigma(G) = 1$ (árvores), $\sigma(G) = 2$ e $\sigma(G) = 3$. Em um trabalho anterior (Costa, D.A.F., 2022), mostramos que um grafo split G tal que $\sigma(G) = 2$ é Classe 2 sse existe $v \in V(G)$, tal que $d(v) = \Delta$ e $H = G[N[v]]$ tal que $|E(H)| \geq \left\lfloor \frac{|V(H)|}{2} \right\rfloor \cdot \Delta$. Neste trabalho, preenchemos mais algumas lacunas no estudo da coloração de arestas dos grafos split através de uma extensão da técnica de coloração de arestas de Plantholt (Plantholt, 1981). Com isso, mostramos que se existe um vértice v em Y tal que $d(v) \leq \frac{\Delta}{2}$ e $v \in N(w)$ para algum $w \in X$ tal que $d(w) = \Delta$, então o grafo split é Classe 1.

On the irregular chromatic index of blow-ups of cycles

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Keywords: Edge-colouring; Irregular chromatic index; Locally irregular graph

A *locally irregular k-coloring* of a graph G is an edge-coloring $\varphi\colon E(G) \to [k]$ of G such that for each color $i \in [k]$, the subgraph of G induced by the edges colored with i is *locally irregular*, i.e. a graph with no adjacent vertices with equal degrees. Baudon *et al.* 2015 described all graphs with no locally irregular colorings. For any other graph G, the *irregular chromatic index* of G, denoted by $\chi'_{\mathrm{irr}}(G)$, is the smallest k such that G admits a locally irregular k-coloring.

Baudon *et al.* 2015 conjectured that $\chi'_{\mathrm{irr}}(G) \leq 3$ for any connected graph G admitting a locally irregular coloring, but it was observed that there exists a graph H with 10 vertices and $\chi'_{\mathrm{irr}}(H) = 4$. The best general upper bound for the irregular chromatic index is 220, due to Lužar 2018, derived from a result on bipartite graphs. It was verified that even length cycles, complete graphs, d-regular graphs with $d \geq 10^7$, and some other classes of graphs have irregular chromatic index at most three.

We are interested in investigating for which graphs G we have $\chi'_{\mathrm{irr}}(G) \leq 2$, and we observe that the problem of determining whether $\chi'_{\mathrm{irr}}(G) \leq 2$ for general graphs G is NP-complete. We say a *blow-up* of a graph H with $V(H) = \{v_1, \ldots, v_h\}$ is a graph G obtained by replacing each vertex v_i of H with an independent set V_i, which we call *class* (distinct classes may have different sizes), such that classes of G associated to adjacent vertices in H induce complete bipartite graphs and there is no other edge in G. We prove the following result, where we note that odd cycles do not admit locally irregular colorings.

Theorem 1. *Let $k \geq 3$ be an integer. If G is a blow-up of a cycle of length k, then $\chi'_{irr}(G) \leq 2$.*

Flow decomposition on arc-coloured networks

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Keywords: splittable flows, arc-coloured digraphs, computational complexity

A *network* $\mathcal{N}$ is formed by a digraph D together with a *capacity function* $u : A(D) \to \mathbb{Z}_+$, and it is denoted by $\mathcal{N} = (D, u)$. A *flow on* $\mathcal{N}$ is a function $x : A(D) \to \mathbb{Z}_+$ such that $x(a) \leq u(a)$ for all $a \in A(D)$, and it is said to be ℓ-splittable if it can be decomposed into up to ℓ paths [Baier, G., Köhler, E. e Skutella, M. (2005)].

Arc-coloured networks are used to model situations where it is crucial to represent qualitative differences among different regions through which the flow will be sent [Granata, D. et al. (2013)]. They have applications in several areas such as communication networks, multimodal transportation, molecular biology, packing, among others.

We say that a flow is λ-uniform if its value on each arc of the network is exactly λ, for some $\lambda \in \mathbb{Z}_+^*$. We consider the problem of decomposing a λ-uniform flow over an arc-coloured network with minimum cost, that is, with minimum sum of the cost of its paths, where the cost of a path is given by its number of colours. This problem can be solved in polynomial time for 2-arc-coloured networks, which are networks with at most 2 colours [Carvalho Neto, C. et al. (2023)]. In this work, we show that it is $\mathcal{NP}$-Complete for 3-arc-coloured networks and for acyclic 5-arc-coloured networks.

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Diameter Reduction Via Flipping Arcs

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Keywords: diameter, diameter reduction, edge flip, arc reversal, edge polytope

The diameter of a directed graph is a fundamental invariant defined as the maximum distance realized among the pairs of vertices. As graphs of small diameter are of interest in many applications, we study the following problem: for a given directed graph and a positive integer d, what is the minimum number of arc flips (also known as arc reversal) required to obtain a graph with diameter at most d? It is a generalization of the well-known problem Oriented Diameter, first studied by Chvátal and Thomassen ("Distances in orientations of graphs", *Journal of Combinatorial Theory, Series B*, 1978).

One motivation of the problem is the following: suppose that there are n airports, some ordered pairs are connected by a flight. For a given integer d, how many flights do we have to establish to make sure that from each airport one can get to another by changing planes at most d times?

We investigate variants of the above problem, considering the number of flips and the target diameter as parameters. We prove that most of the related questions of this type are hard. Special cases of graphs are also considered, such as planar and cactus graphs, where we give polynomial time algorithms.

Session 6

Inclusion graphs of biclique parts of K_3–free graphs

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Keywords: Biclique graph; Partially ordered set; Comparability graph; K_3–free graph

A *biclique* in a graph G is a maximal subset of its vertices that induces a complete bipartite graph. Each part of the bipartition is called a *biclique part* of G. Let $\widehat{\mathcal{B}}_G$ be the set of all biclique parts of G. In this work we introduce the *inclusion graph of biclique parts* of a graph G, which is denoted as $\mathrm{BP}(G)$, as the graph that contains $\widehat{\mathcal{B}}_G$ as vertices and the edges correspond to the inclusion order of biclique parts.

We also introduce a subclass of comparability graphs called Skew-IIC — for *Skew-symmetric, corresponding predecessor-successor comparison free, IIC-comparability graphs* — and show that the class of inclusion graphs of biclique parts of K_3–free graphs is the same as the class of Skew-IIC graphs, from which we derive a characterization of biclique graphs of K_3–free graphs.

From this result, we also present a subclass of K_3–free graphs where the class of their biclique graphs is the same as the class of biclique graphs of all K_3–free graphs. The graphs in this subclass have some interesting properties such as admitting a perfect matching such that each edge of this matching is contained in only one biclique — and every biclique contains one of such edges — and that their mutually included biclique graphs are induced subgraphs of the square of their line graphs.

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Erdős sparse halves conjecture for blow-ups of Vega graphs

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Keywords: Triangle-free graphs; Sparse halves; Vega graphs;

Erdős (1975) conjectured that every triangle-free graph with n vertices contains a *sparse half*, which is a subset of $\lfloor n/2 \rfloor$ vertices that induces at most $n^2/50$ edges. Krivelevich (1995) verified this conjecture for triangle-free graphs with n vertices and minimum degree at least $2n/5$. Keevash and Sudakov (2006) improved this result by extending it to triangle-free graphs with average degree at least $2n/5$.

Norin and Yepremyan (2015) verified the conjecture for graphs G with $\delta(G) \geq 5n/14$ and average degree at least $(2n/5 - \gamma)n$ for any $\gamma > 0$. Recently, Bedenknecht, Mota, Reiher, and Schacht (2019) confirmed the conjecture for graphs homomorphic to *Andrásfai graphs*, from which it follows that the conjecture is valid for triangle-free graphs with minimum degree greater than $10n/29$. Furthermore, if the chromatic number of the graph is less than 4, then the minimum degree condition can be relaxed to greater than $n/3$.

It is known that if Erdős' conjecture holds for graphs homomorphic to some *Vega graphs*, then it also holds for triangle-free graphs G with $\delta(G) > n/3$. We verify Erdős' conjecture to graphs homomorphic to any graph $H_x = H - x$ obtained from any Vega graph H by removing some vertex x of H. We construct a sequence of graphs $G \coloneqq G_1, G_2, \ldots, G_k$ such that if G_{i+1} has a sparse half, then so does G_i. Furthermore, we guarantee that G_k is a subgraph of a blow-up of an Andrásfai graph. The theorem follows from the result of Bedenknecht, Mota, Reiher, and Schacht.

Estudo sobre 3-atribuição de papéis para produtos direto de grafos

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Keywords: atribuição de papéis, produto direto, redes sociais

A inteligência artificial desempenha um papel crucial na sociedade contemporânea, possibilitando a construção de perfis complexos a partir de vastos conjuntos de dados. Estes perfis, por sua vez, podem ser representados como grafos, sendo assim, certas características podem se relacionar a certos interesses, como idade, gênero e profissão. Nesse aspecto, o conceito de atribuição de papéis, introduzido por Everett e Borgatti (1991), pode ter um papel fundamental ao buscar e comparar estes perfis, já que pode reduzir a grande quantidade de características para estruturas mais compactas. Uma *r-atribuição de papéis* de um grafo simples G é um homomorfismo localmente sobrejetor, ou seja, é um mapeamento dos vértices de r para G de modo que a relação de vizinhança seja preservada. Além disso, uma r-atribuição de papéis específica define um grafo de papéis, no qual os vértices são os r papéis distintos, e existe uma aresta entre dois papéis sempre que há dois vértices relacionados no grafo G que correspondem a esses papéis. Atribuição de papéis tem sido um tema pouco explorado em produtos de grafos, em particular em produto direto. Dados dois grafos G e H, o *produto direto* $G \times H$ é o grafo com conjunto de vértices resultantes do produto cartesiano de $V(G)$ e $V(H)$ e o conjunto de arestas $\{(u,u')(v,v') \mid uv \in E(G),\ u'v' \in E(H)\}$. Neste trabalho, demonstramos que se G e H são grafos bipartidos não triviais, então $G \times H$ sempre terá uma 3-atribuição de papéis. Também demonstramos que se G possui uma R-atribuição de papéis e H possui uma S-atribuição de papéis, então $G \times H$ possui uma $(R \times S)$-atribuição de papéis.

Transversals of longest paths on cubic pseudo-Halin graphs

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Keywords: Halin graph, pseudo-Halin graph, transversal, longest paths

A Halin graph is a plane graph $G = T \cup C$, where T is a plane tree with at least four vertices and no vertex of degree two, and C is a cycle connecting the leaves of T in the cyclic order determined by the embedding of T in the plane. These graphs were first studied by Halin [1]. Bondy and Lovász proved in 1985 (a conjecture announced in [1]) that they are pancyclic if T has no vertex of degree 3. We consider Halin-like simple graphs $G = T \cup H$, in which H is a 2-factor on the leaves of T, but G is not necessarily planar. We call such graphs pseudo-Halin graphs.

We are interested in the minimum size of transversals of all (resp. three) longest paths on cubic pseudo-Halin graphs. For a graph G, we use the notation $\mathrm{lpt}(G)$ (resp. $\mathrm{lpt}_3(G)$) to refer to this parameter. The motivation for this study is the fact that it has been conjectured that $\mathrm{lpt}_3(G) = 1$ for all connected graphs, but this is not known to hold even for cubic graphs. For general connected graphs G, it is known that $\mathrm{lpt}(G)$ is not always 1.

We investigate first the case in which the tree T has a root of degree three and the subtrees of the root are perfect binary trees of the same height. We refer to this class of graphs as *cubic balanced pseudo-Halin* graphs. We do not have results for all such graphs, but we show that $\mathrm{lpt}(G) = 1$ (or $\mathrm{lpt}_3(G) = 1$) for some classes of 2-factors H. Moreover, we prove that some of the results for the balanced case also hold when the three binary trees may have different heights.

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Session 7

Sufficient conditions that ensure trees to have game chromatic number 4 ‡

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Keywords: Coloring game, Vertex coloring, Combinatorial games.

The *coloring game* is a two-player non-cooperative game conceived by Steven Brams, first published by Gardner in 1981. Alice and Bob alternate turns to properly color the vertices of a finite graph G with t colors. Alice's goal is to properly color the vertices of G with t colors; Bob's aim is to prevent it. If at any point there's an uncolored vertex without an available color, Bob wins; otherwise, Alice wins. The game chromatic number $\chi_g(G)$ of G is the smallest t for Alice to have a winning strategy. In 1991, Bodlaender showed the smallest tree T with $\chi_g(T) = 4$; Faigle et al. (1993) proved $\chi_g(T) \leq 4$ for every tree T. Later, Dunn et al. (2015) proposed characterizing forests with game chromatic numbers 3 and 4 as an open problem.

The motivation behind this research stems from the examination of the coloring game played on 1-caterpillars as introduced by Guignard (2010), where the characterization of the game for such graphs was proposed. We verify Guignard's results for the cases when Bob wins the game with 3 colors in 1-caterpillars, and extend them for trees containing these caterpillars, obtaining sufficient conditions for a tree to have game chromatic number 4.

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Some results on graph coloring games

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Keywords: Combinatorial games, graph coloring, split graphs, threshold graphs, asymmetric coloring, game chromatic number

In the *graph coloring game*, given a graph G and a set C of colors, two players, Alice and Bob, take turns, starting with Alice, by choosing an uncolored vertex and assigning to it a color of C. The chosen color must not be already assigned to any of its neighbours [1]. The game ends when there are no valid plays. Alice wins if and only if all vertices are colored.

In the *asymmetric* version, the (a-b)-coloring game, Alice colors a vertices on their turn and Bob colors b vertices [2]. A *two turn game* is an (a-b)-coloring game with $a+b \geq |V(G)|$, and we define it as $(a, |V(G)|-a)$-coloring game.

The *game chromatic number* $\chi_g(G)$ is the least integer k which if $|C| = k$ then Alice has a winning strategy in the coloring game on G.

We show that if G is a threshold graph with maximum clique of size $k \geq 3$, then $\chi_g(G) \leq 2k - 3$, and if G is a split graph with maximum clique of size k, then $\chi_g(G) \leq 2k - 1$, and that both of those bounds are tight.

We also show an polynomial time algorithm to decide whether Alice has a winning strategy on the $(k, |V(G)| - k)$-coloring game, for any fixed k.

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Results on the connected greedy coloring game[1]

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Keywords: coloring game, graph coloring, game connected grundy number

The graph coloring game is a maker-breaker game proposed by Bodlaender in 1991 that consists in, given a graph G and a set of colors C, the two players, Alice and Bob, take turns selecting a vertex v from G and choosing a color c from C for v, ensuring that none of v's neighbors are already colored with c, that is, the partial coloring of the graph must be a proper coloring. Alice, which is the first one to play, wins if all vertices in G are colored and Bob wins if he can prevent that. Two variants of the original game were proposed, the greedy variant, which was proposed by Havet and Zhu in 2013, and the connected variant, which was proposed by Charpentier et al. in 2020. In the greedy variant of the game, $C = \{1, \ldots, k\}$ and each player chooses only the vertex v they want to color, the color assigned to v is the highest integer $1 \leq x \leq k$ such that, for every color less than x, there exists a neighbor of v with that color. In the connected variant, the subgraph induced by the colored vertices must always be connected and, because of that, the graph itself must be connected. All our contributions are about the greedy connected variant of the coloring game, which is the variant we get by combining the restrictions of both variants. It was introduce in (Lima et al. The connected greedy coloring game, Theor. Comput. Sci., **940**, 1–13, 2023), where they defined $\Gamma_{cg}(G)$ as the least integer k for which Alice has the winning strategy when $C = \{1, 2, \ldots, k\}$. We proved that $\Gamma_{cg}(G) = 2$ if and only if G is a bipartite graph with at least one edge and that $\Gamma_{cg}(G) \leq 1 + \max_{B \in \mathcal{B}(G)} \Delta(B)$, where $\mathcal{B}(G)$ is the set of all blocks of G. As a consequence of these two results, we can conclude that, if G is a cactus with at least one edge, $2 \leq \Gamma_{cg}(G) \leq 3$, where $\Gamma_{cg}(G) = 3$ if and and only if G is not a bipartite graph.

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Session 8

The Path Validity Problem

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Keywords: Labelling, Path Validity Problem, $\mathcal{NP}$-complete problem.

The labelling of a graph involves assigning labels to its vertices and/or edges under specific constraints. A large and relevant number of studies on this topic has been published up until the year 2022, with most of them listed in the dynamic survey organized by Gallian [2]. The *Path Validity Problem* [1] deals with finding an integer vertex numbering that induces the maximum or minimum quantity of valid paths, i.e., 2-paths with the middle vertex label smaller than those at the endpoints.

Let $G = (V(G), E(G))$ be a simple connected graph with $n = |V(G)|$. A numbering π of G is a bijective mapping from $V(G)$ to $\{1, 2, \ldots, n\}$, with $\pi(v)$ denoting the number associated with vertex $v \in V(G)$. The validity $\phi_\pi(G)$, of a numbering π of G, is the quantity of valid 2-paths of G induced by π. The *minimum (maximum) validity* $\phi_{\min}(G)$ ($\phi_{\max}$) of G is the minimum (maximum) of $\phi_\pi(G)$ over all possible numberings of G. $\phi_{\min}(G)$ ($\phi_{\max}$) can also be stated as the decision problem MIN-VP(G, k) (MAX-VP(G, k)), which asks "Is there a numbering π of G such that $\phi_\pi(G) \leqslant k$ ($\phi_\pi(G) \geqslant k$)?".

This work presents the main results recently achieved in [3]. Those results include a proof that MAX-VP(G, k) is an $\mathcal{NP}$-complete problem (even in regular graphs of degree 3 (cubic graphs)), and the behaviour of the $\phi_{\min}(G)$ and $\phi_{\max}(G)$ functions in some other graph classes.

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On Finding Temporal Cycles and Acyclic Labelings

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Keywords: Temporal graphs; Cycles; Network design.

Temporal graphs [1], representing dynamic interactions among entities over time, have become pivotal in modeling various real-world systems. Unfortunately though most problems become hard on these structures, and in particular connectivity problems[2] and connected components[3]. Lately, also network design problems have been of interest[4].

In this paper, we investigate the fundamental concept of cycles on temporal graph. As it usually happens in temporal graphs, a notion on static graphs can have many possible dynamic interpretations. We introduce the following notions. Given a temporal graph (G, λ) and a cycle C in G, we say that C is a *temporal cycle* of type: *one* if $\exists x \in V(C)$ such that the edges of C contain a non-trivial *temporal* x, x*-path* (path from x to x whose edge labels are non-decreasing); *paired* if the edges of C contain a temporal x, y-path and a temporal y, x-path, for some pair $x, y \in V(C)$; and *all* if the edges of C contain a non-trivial temporal x, x-path $\forall x \in V(C)$. For each type of cycle, we investigate the problems of detecting a cycle and of designing a "acyclic" temporal graph. We give many polynomial and hardness results. In particular, building a temporal graph with no type one cycle is NP-complete for fixed lifetime and can always be done if the graph has no C_4's.

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Inaproximabilidade do k-center justo

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Keywords: k-center justo, inaproximabilidade

Entre os problemas de *clustering* mais importantes, está o k-center: dado um grafo $G = (V, E)$ e um número k, queremos encontrar $S \subseteq V$ com $|S| \leq k$ e uma atribuição $\phi : V \to S$ que minimiza $\max_{u \in V} d(u, \phi(u))$, onde $d(u, v)$ é a distância entre u e v. É bem conhecido que esse problema admite uma 2-aproximação, que é ótima a não ser que P = NP (Gonzalez, 1985).

Em aplicações cujos pontos estão associados a dados de origem, o particionamento obtido por meio de problemas clássicos pode levar a injustiças, já que subgrupos podem ficar sub-representados em algumas das partes. Chierichetti et al. (NIPS 2017) consideraram uma versão balanceada do problema, que se restringe a soluções *justas*. Nessa variante, supomos que V é a união de ℓ subconjuntos, $V_1, \ldots, V_\ell$, representando cores associadas aos vértices. Note que um vértice pode ter mais de uma cor. Uma solução é *justa* se, para cada centro $v \in S$ e cada cor $i = 1, \ldots, \ell$, tivermos

$$\frac{|\phi^{-1}(v) \cap V_i|}{|\phi^{-1}(v)|} = \frac{|V_i|}{|V|}.$$

Chierichetti et al. deram uma 3-aproximação para a versão com duas cores e em que cada vértice tem apenas uma cor e questionaram se o problema admitiria uma aproximação melhor. Posteriormente, Bercea et al. (APPROX 2019) forneceram uma 5-aproximação para o caso com várias cores. Além disso, também apresentaram uma redução para tentar demonstrar que o problema não admitiria uma aproximação com fator melhor do que 3.

Neste trabalho, observamos que a redução de Bercea et al. está incorreta e apresentamos uma nova redução para demonstrar que, de fato, a versão justa do problema e com pelo menos 32 cores não admite uma $(3 - \varepsilon)$-aproximação, para $\varepsilon > 0$, a não ser que P = NP. Deixamos em aberto determinar se o caso particular do problema com menos cores ou em que cada vértice tem apenas uma cor admite uma 2-aproximação.

Session 9

Infinite families of two Kochol superpositions of Loupekine snarks are Type 1 ‡

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Keywords: total coloring, snarks, TCC.

A *q-total coloring* of a graph G is an assignment of q colors to the vertices and edges of G, so that adjacent or incident elements have different colors. The *Total Coloring Conjecture* (TCC) asserts that the total chromatic number of G is at least $\Delta+1$, but at most $\Delta+2$ (Behzad (1065), Vizing (1964)). This conjecture led to the classification of graphs into two types: *Type 1*, if $\chi'' = \Delta + 1$, and *Type 2*, if $\chi'' = \Delta + 2$.

While the TCC has been verified for specific graph families, it remains an open problem for many graph classes, spanning over five decades. For cubic graphs G, Rosenfeld (1971) and Vijayaditya (1971) independently established that $4 \leq \chi''(G) \leq 5$.

In 2003, Cavicchioli et al. showed, with the assistance of a computer, that all *snark* (connected bridgeless cubic graphs whose edges cannot be properly coloured with three colours) with *girth* (length of the shortest cycle contained in G) at least 5 and fewer than 30 vertices are Type 1, and asked the following question: "Find (if any) the smallest snark (with respect to the order) which is of Type 2". In 2015, Brinkmann, Preissmann and D. Sasaki divided this problem in two questions to investigate the true obstruction that makes finding these graphs (if they exist at all) challenging: either being snarks or having a girth at least 5.

In this work, we determine that all members of new infinite families of snarks obtained by the Kochol superposition of Loupekine with Blowup and Semiblowup snarks are Type 1. These results contribute to the question posed by Brinkmann et al. (2015) by presenting negative evidence about the existence of Type 2 cubic graphs with girth at least 5.

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On the AVD-total chromatic number of 4-regular circulant graphs

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Keywords: : Adjacent-vertex-distinguishing total coloring, adjacent-vertex-distinguishing total chromatic number, circulant graphs

An *AVD-k-total coloring* of a simple graph G is a mapping $\pi : V(G) \cup E(G) \to \{1, \ldots, k\}$ such that: adjacent or incident elements $x, y \in V(G) \cup E(G)$, $\pi(x) \neq \pi(y)$; and for each pair of adjacent vertices $x, y \in V(G)$, sets $\{\pi(x)\} \cup \{\pi(xv) \mid xv \in E(G) \text{ and } v \in V(G)\}$ and $\{\pi(y)\} \cup \{yv \in E(G) \text{ and } v \in V(G)\}$ are distinct. The *AVD-total chromatic number*, denoted by $\chi''_a(G)$, is the smallest k for which G admits an AVD-k-total-coloring. In 2005, Zhang et al. [1] conjectured that any graph G has $\chi''_a(G) \leq \Delta + 3$, where Δ is the maximum degree of G and this conjecture is known as AVD-Total Coloring Conjecture (AVD-TCC). Since then, recent studies have been developed involving some graph classes to investigate the AVD-TCC, such as equipartite graphs, split graphs, corona graphs, and 4-regular graphs. If AVD-TCC holds, we can classify any graph according to the AVD-total chromatic number. If $\chi''_a(G) = \Delta + 1$, then G is called *AVD-Type 1*. If $\chi''_a(G) = \Delta + 2$, then G is called *AVD-Type 2*. If $\chi''_a(G) = \Delta + 3$, then G is called *AVD-Type 3*. A *circulant graph* $C_n(d_1, d_2, \cdots, d_\ell)$ with integers numbers $1 \leq d_i \leq \lfloor n/2 \rfloor$, where $1 \leq i \leq \ell$ and $\ell \leq \lfloor n/2 \rfloor$, has vertex set $V = \{v_0, v_1, \cdots, v_{n-1}\}$ and edge set $E = \bigcup_{i=1}^{\ell} E_i$, where $E_i = \{e_0^i, e_1^i, \cdots, e_{n-1}^i\}$ and $e_j^i = v_j v_{j+d_i}$ (if n is even and $d_\ell = n/2$, then $E_\ell = \{e_0^\ell, e_1^\ell, \ldots, e_{\frac{n-2}{2}}^\ell\}$), where the indexes of the vertices are considered modulo n. An edge of E_i has length d_i. Some classical circulant graphs, such as the cycle graphs $C_n \simeq C_n(1)$ and the complete graphs $K_n \simeq C_n(1, 2, ..., \lfloor n/2 \rfloor)$ have their AVD-total chromatic number determined. In this work, we prove that the 4-regular circulant graph $C_n(1, k)$, for n and $\ell = \frac{n}{\gcd(n,k)}$ even, is *AVD-Type 2*.

The AVD-total chromatic number of fullerene nanodiscs

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Keywords: AVD-total coloring, cubic graphs, fullerene nanodiscs

An *AVD-k-total coloring* of a simple graph G is a mapping $\pi : V(G) \cup E(G) \rightarrow \{1, \ldots, k\}$, with $k \geq 1$ such that: for each pair of adjacent or incident elements $x, y \in V(G) \cup E(G)$, $\pi(x) \neq \pi(y)$; and for each pair of ajdacent vertices $x, y \in V(G)$, sets $\{\pi(x)\} \cup \{\pi(xv) \mid xv \in E(G)$ and $v \in V(G)\}$ and $\{\pi(y)\} \cup \{yv \in E(G)$ and $v \in V(G)\}$ are distinct. The *AVD-total chromatic number*, denoted by $\chi''_a(G)$, is the smallest k for which G admits an AVD-k-total-coloring. It is conjectured that if G is a simple graph, then $\chi''_a(G) \leq \Delta + 3$, where Δ is the maximum degree of G, and this conjecture is known as AVD-TCC. If AVD-TCC holds, then we can classify any graph according to the AVD-total chromatic number. If $\chi''_a(G) = \Delta + 1$, then G is called *AVD-Type 1*. If $\chi''_a(G) = \Delta + 2$, then G is called *AVD-Type 2*. If $\chi''_a(G) = \Delta + 3$, then G is called *AVD-Type 3*. In 2008, Hulgan conjectured that if G is a simple graph with $\Delta = 3$, then $\chi''_a(G) \leq 5$. In 2017, Luiz et al. [1] proved that the Hulgan's conjecture holds for infinite families of snarks. Fullerene nanodiscs D_r are mathematical models for carbon-based molecules experimentally found in the early eighties, which are cubic, 3-connected, planar graphs with pentagonal and hexagonal faces. The planar embedding of D_r has its faces arranged into layers, one layer next to the nearest previous layer starting from a hexagonal cover until we reach the other hexagonal cover. The distance between the inner (outer) layer and the central layer, where lie 12 pentagonal faces, is given by the radius parameter $r \geq 2$. In this work, we prove that the fullerene nanodisc $D_r, r \geq 2$ is *AVD-Type 2*, giving a positive evidence to Hulgan's Conjecture.

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Equitable Total Coloring of Grid Graphs[1]

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Keywords: equitable total coloring, total coloring, grid graphs

A proper total coloring of a graph G is an assignment of colors to the vertices and edges of G, ensuring that any two adjacent or incident elements (vertices and/or edges) are not colored the same. The minimum number of colors for a total coloring of G is the total chromatic number, denoted $\chi''(G)$. An equitable total coloring of G is a proper total coloring where the cardinality of any two color classes differs by at most one. The minimum number of colors required for an equitable total coloring of G is the equitable total chromatic number, denoted by $\chi''_e(G)$. The Cartesian product of two disjoint graphs G and H is the graph $G \times H$ with vertex set $V(G) \times V(H)$ where (u_1, v_1) is adjacent to (u_2, v_2) whenever $u_1 = u_2$ and v_1 is adjacent to v_2; or $v_1 = v_2$ and u_1 is adjacent to u_2. The grid graph is the Cartesian product $P_m \times P_n$, where P_m and P_n are the paths with m and n vertices, respectively. The total chromatic number of grid graphs was determined by Campos in 2006 [1]. Grid graphs are a subclass of bipartite graphs, the graphs whose vertex set can be partitioned into two independent sets. Fu determined the equitable total chromatic number of complete bipartite graphs [2]. However, to the best of our knowledge, the equitable total chromatic number of grid graphs had remained unknown. Now, we prove $\chi''_e(G) = \chi''(G)$ and our proof leads to a polynomial time algorithm for an optimum equitable total coloring of grid graphs.

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Session 10

Algoritmos exatos para o problema do Número de Grundy

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Palavres-chave: Teoria dos grafos. Coloração de grafos. Número Grundy. Algoritmos.

Dado um grafo G = (V(G),E(G)), o Problema do Número de Grundy consiste em determinar a maior quantidade de cores que o Algoritmo Guloso de Coloração consegue atribuir a um grafo. Este algoritmo tem o seguinte princípio geral: receber, um a um, os vértices do grafo a ser colorido, atribuindo sempre a menor cor possível a este vértice. O maior número de cores utilizadas pelo Algoritmo Guloso de Coloração é chamado de número de Grundy ou número cromático guloso do grafo e é denotado por $\Gamma(G)$, no qual fornece um limite de pior caso para o conhecido problema da coloração de vértices. Determinar o número de Grundy de um grafo qualquer pertence à classe NP-completo e dada a complexidade do problema, diversos trabalhos na literatura apresentam limitantes para $\Gamma(G)$ ou determinam o valor de $\Gamma(G)$ para classes de grafos.

Neste trabalho, apresentamos soluções exatas para o problema proposto. Desenvolvemos dois algoritmos exatos utilizando, respectivamente, Programação Dinâmica e Branch and Bound. Propomos também tanto um limite inferior quanto um superior, ambos baseados em heurísticas gulosas. Além disso, apresentamos três formulações de Programação Linear Inteira. Para avaliar as soluções algorítmicas, geramos instâncias aleatórias e da classe Stacked Book, sobre as quais realizamos diversos testes computacionais, comparando-os com base no tempo de execução.

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Scheduling Transmissions in Time-Slotted LoRa Wide Area Networks

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Keywords: Optimization, LoRaWAN, Scheduling, Complexity

LoRa is a low-power and long range radio communication technology designed for low-power Internet of Things devices. The devices of a LoRa wide area network (LoRaWAN) are often deployed in remote areas where the end-to-end connectivity provided through one or more gateways may be limited. When the gateway is not available at all times, the sensing data need to be buffered locally and transmitted as soon as a gateway becomes available. To avoid bursts of collisions in a classical random channel access method like Aloha, we propose to introduce a time-slotted transmission scheduling mechanism.

LoRaWAN mainly uses frequency bands for which network usage is limited to meet the requirements of mass deployment of several thousand connected objects. For example, in the European Union, a device cannot use frequency channels for more than 1% of the time in its duty cycle. This leads to a scheduling problem with waiting times. The quality of communications between connected devices and gateways in LoRaWAN depends on a set of parameters, one of the main ones having a direct impact on overall throughput is the spreading factor (SF). The TS-LoRa protocol proposes an allocation of spreading factors for the communications of end-devices, over different periods, to respect these constraints. Here, we focus on a scheduling problem within this framework.

In this work, we formalize the LoRaWAN data scheduling optimisation problem with waiting times using linear programming. We derive an optimal algorithm for the special case of equivalent SF. We then investigate approximation algorithms for specific and general cases. We finally discuss about the general problem's complexity.

Graphical Traveling Salesman Problem in some graph classes

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Keywords: TSP, release dates, complexity, graph classes

The Graphical Traveling Salesman Problem with release dates (GTSP-rd) is a relaxation of the Traveling Salesman Problem with release dates (TSP-rd) (Archetti et al., 2015) in which the input to the problem don't need to bee a complete graph and in this case a *route* is a walk in G, that must be traversed by the Traveling Salesman, some for delivery and others just to enable it. The route always starts and ends at the depot. Formally, $\mathcal{R} = \{(i_0, b_0), (i_1, b_1), \ldots, (i_s, b_s), (i_{s+1}, b_{s+1})\}$ with $i_0 = i_{s+1} = 0$ and $b_0 = b_{s+1} = 0$. The set $S = \{i_1, \ldots, i_s\} \subseteq N$ and $(i_k, i_{k+1}) \in E$. The vertices within S are divided into two sets, the vertices where deliveries are made on the route, the *delivery vertices*, $S^d = \{i_k \in S \mid b_k = 1\}$. The other vertices are called *traverse vertices*, $S^t = S \setminus S^d$.

A *solution* to GTSP-rd consists of a set of routes $\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_x$ containing the vertices set $S_1, S_2, \ldots, S_x$, these routes must be done consecutively by the Traveling Salesman in order of dispatch time. That is, given the dispatch time of a route $T_{\mathcal{R}} \geq \max_{v \in S^d}\{r_v\}$ then $T_{\mathcal{R}_1} < T_{\mathcal{R}_2} < \cdots < T_{\mathcal{R}_x}$. A route i can only leave the depot if the previous route has already be attended, that is, $T_{\mathcal{R}_{i-1}} + d_{\mathcal{R}_{i-1}} \leq T_{\mathcal{R}_i}$. A solution to GTSP-rd is feasible if all the set vertices $S_i^d \subseteq S_i$ form a partition of N. In this work, we address two objective functions: minimizing time completation (GTSP-rd(time)) and minimizing total distance traveled by routes (GTSP-rd(distance)).

As done in Cornuéjols et al. (1985) and Fonlupt and Nachef (1993) for the GTSP as a relaxation of TSP, we aim to explore the GTPS-rd within special graph classes, such as paths, stars and cycles, discerning the levels of complexity and identifying potential avenues for efficient solutions.

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Algorithms for the Maximum Clique Problem with Bit-Level Parallelism

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Keywords: Graphs; Maximum Clique; Coloring; Roaring bitmaps; Branch and Bound.

This work addresses the Maximum Clique Problem (MCP) in graphs, a well-known challenge in graph theory and combinatorial optimization. Most state-of-the-art branch-and-bound (B&B) algorithms use greedy coloring heuristics to determine both branching order and upper bounds. In many cases, differences in computational environments can make comparisons less precise. In this study, we perform a computational comparison of the MCS algorithm, presented in the unified framework for MCP by [2], with two bit-parallel algorithms using the Roaring Bitmap structure. This structure has proven efficient in leveraging bit parallelism as proposed for more recent algorithms. This technique leverages the efficiency of bit-level parallelism to accelerate operations such as intersection and union of vertex sets, reducing the execution time in branch-and-bound B&B algorithms.

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Session 11

The Firefighter problem on Loupekine snarks

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Keywords: Firefighter problem, Loupekine snarks, cubic graphs, bandwidth

The *Firefighter problem* consists of a fire starting at one vertex of a graph and then a non-burned vertex is chosen to be defended, making unburnable. At each new step, the fire spreads to all adjacent vertices that were not defended in previous steps and, again, one vertex can be defended by firefighters, and this process continues until the fire stops spreading [1]. Let $sn(G, v)$ denote the maximum number of vertices that can be saved when a fire breaks out at vertex v of graph G. The surviving rate $\rho(G)$ is the expected number of vertices that can be saved if the initial vertex is chosen uniformly at random.

In 2015, Costa established the surviving rate for some snarks [2]. With this motivation, we investigate the family of *Loupekine snarks*. We develop a strategy in which the number of burned vertices is bounded by a constant throughout both families. Then, we generalize this result by showing that for any graph G with bandwidth at most b, there is a strategy where the number of burned vertices is $O(b^2)$.

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Graph aspects and algorithms of a Tower of Hanoi-London hybrid game[1]

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Keywords: algorithms, graph properties, mathematical game, tower of Hanoi.

The Tower of Hanoi-London (ToHL) is a puzzle which combines both the rules of the Tower of Hanoi (ToH) and the constraints of the Tower of London (ToL). It consists of n discs $d_1, d_2, \ldots, d_n$ of different sizes randomly stacked on 3 pegs p_0, p_1, p_2. The discs on each peg, from bottom to top, are sorted in decreasing order of size and p_0, p_1, p_2 having maximum capacities of $n, n-1, n-2$, respectively. In this puzzle, the goal is to move all discs to p_0, respecting capacity constraints and the rules of the ToH. Namely, only one disc can be moved at a time, and disc d_i can be placed on d_j only if $d_i < d_j$. The graph HL_n has all possible states of the ToHL puzzle as vertices and possible moves between the states as edges. This work shows a characterization of HL_n based on a recursive definition of the graph of the ToH H_n. The last has a triangular shape with three special vertices: the top, left, and right extremities, that correspond to the states where all discs are on a single peg. In addition, it is shown that HL_n has two components, and the vertices of V are partitioned into V_g and V_b, where V_g are the good vertices and V_b are the bad vertices. The graph HL_n has $3^n - 2(n+1)$ vertices. Where, $|V_g| = \lfloor \frac{3^n}{2} \rfloor - n$ and $|V_b| = \lfloor \frac{3^n}{2} \rfloor - (n+1)$. Partitioning HL_n into subgraphs TOP, ESQ, and DIR following the recursive scheme of H_n, all the following assertions are valid: i) TOP is isomorphic to H_{n-1} minus the left extremity; ii) ESQ is isomorphic to H_{n-1} minus the top and left extremities. iii) DIR is isomorphic to HL_{n-1} by adding the leftmost extremity. Also, an algorithm is presented to decide whether a given state of the ToHL is good or bad, i.e., whether it is possible to reach the goal from a certain state or not, as well as an algorithm to solve the game when possible. The algorithm to solve the game has a time complexity of $O(2^n)$, similar to the ToH.

[1] Authors are from ALGOX (Algorithms, Optimization and Computational Complexity) research group and the Postgraduate Program in Informatics (PPGI/UFAM). Partial support by the Coord. for the Improvement of Higher Education Personnel (CAPES-PROEX) - 001, National Council for Scientific and Technological Development (CNPq), and the Amazonas State Research Support Foundation (PDPG CAPES/FAPEAM - Edital N° 038/2022 and FAPEAM-POSGRAD - 2024-2025).

The Harmonious Colouring Game[1]

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Keywords: Colouring Game, Harmonious Colouring, Complexity

A harmonious k-colouring of a n-vertex graph $G = (V, E)$ is a 2-distance proper k-colouring of its vertices such that each edge is uniquely identified by the colours of its endpoints. That is, this is a function $c : V \to [1, k]$ such that, for any two vertices $u \neq v$ at distance at most 2, $c(u) \neq c(v)$, and, for any $a, b \in [1, k]$, there exists at most one edge $\{u, v\} \in E$ such that $c(u) = a$ and $c(v) = b$. Harmonious colouring has been introduced in [2] and studied, e.g., in [3]. Since their introduction by Bodlaender [1], colouring games have been widely studied [4]. Here, we introduce the harmonious colouring game.

In this 2-Player game, alternately, first Alice and then Bob, picks an uncoloured vertex and assigns to it a colour $c(v) \in [1, k]$ with the constraint that, at every turn, the set V' of coloured vertices must induce a valid partial harmonious colouring game. That is, there must exist a harmonious n-colouring c' such that $c'_{|V'} = c$ (i.e., c can be extended to a harmonious n-colouring). Alice wins if, eventually, all vertices are coloured and Bob wins otherwise, i.e., if at some turn, no vertex can be coloured (using the available k colours) still satisfying the constraint. The harmonious game chromatic number $h_g(G)$ of a graph G is the minimum integer k such that Alice has a winning strategy on G with k colours.

In this ongoing work, we first show that $4\left\lfloor \frac{n}{12} \right\rfloor + 1 \leq h_g(P_n) \leq \frac{2(n+3)}{3} + 1$ for any n-node path P_n (similar bounds hold for n-vertex cycles). Then, we also give lower and upper bounds on $h_g(S)$ for any forest S of stars. We also prove that if the players are required to colour the vertices in a given order or if some of the vertices are pre-coloured then it is PSPACE-complete to decide if Alice has a winning strategy with k colours even when the game is played on a split graph.

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Session 12

$[k]$-Roman Domination on Triangulated Discs*

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Keywords: [k]-Roman domination, planar graph, triangulated disc

Let $G = (V, E)$ be a simple graph. As usual, the set of neighbors of $v \in V$ is denoted by $N(v)$. The *closed neighborhood* of $v \in V$ is $N[v] = \{v\} \cup N(v)$. Let k be a positive integer and f be a function that assigns *labels* from the set $\{0, 1, \ldots, k+1\}$ to the vertices of G. The *active neighborhood* $AN(v)$ of a vertex $v \in V$ with respect to f is the set of all neighbors of v that have positive labels. A *[k]-Roman domination function* ($[k]$-RDF) on G is a function $f\colon V(G) \to \{0, 1, ..., k+1\}$ such that, for every vertex $v \in V(G)$ with label $f(v) < k$, $\sum_{u \in N[v]} f(u) \geq |AN(v)| + k$. The *weight* of a $[k]$-RDF f on G is the sum of the labels of the vertices of G under f. The $[k]$-*Roman domination number* $\gamma_{[kR]}(G)$ of G is the minimum weight of a $[k]$-RDF on G. The concept of $[k]$-RDF was introduced by Ahangar et al. in 2021 and it is motivated by a military problem. $[k]$-Roman domination functions were first investigated for the case $k = 1$ by Cockayne et al. in 2004 under the name of Roman Domination. Later, in 2016, Beeler et al. studied the case $k = 2$, under the name of double Roman domination. In 2023, Khalili et al. proved that, given a graph G and a positive integer ℓ, deciding whether G has a $[k]$-RDF with weight at most ℓ is an NP-complete problem.

A classic family of graphs are the planar graphs. A graph is *planar* if it can be drawn on the plane without crossing edges; when it is drawn on the plane satisfying this property, it is called a *plane graph*. An important subclass of planar graphs are *triangulated discs*, that are plane graphs on which all of their inner faces are triangles. In this work, we study $[k]$-Roman domination on subclasses of triangulated discs. We determine the exact value of $\gamma_{[kR]}(G)$ for second powers of paths when $1 \leq k \leq 2$. For $k \geq 1$, we present upper bounds for $\gamma_{[kR]}(G)$ of triangulated web graphs, triangular grids, and also for maximal outerplanar graphs without internal triangles and, in this case, we show that the upper bound is tight.

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Double Roman Domination on graphs with maximum degree 3*

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Keywords: domination in graphs, double Roman domination, cubic graphs

A *Double Roman Dominating Function* (DRDF) on a graph G with vertex set $V(G)$ and edge set $E(G)$ is a function $f\colon V(G) \to \{0,1,2,3\}$ such that if $f(v) = 0$, then vertex v must have at least two neighbors assigned 2 under f or one neighbor u with $f(u) = 3$, and if $f(v) = 1$, then v must have at least one neighbor u with $f(u) \geq 2$. An *independent double Roman dominating function* (IDRDF) f on a graph G is a double Roman dominating function such that the set formed by the vertices with labels different from 0 form an independent set on G. The *weight* of f is $\sum_{v\in V(G)} f(v)$. The minimum weight of a DRDF (IDRDF) on G is the *double Roman dominating number* (*independent double Roman dominating number*) of G, denoted by $\gamma_{dR}(G)$ ($i_{dR}(G)$). The (independent) double Roman domination problem consists in determining $\gamma_{dR}(G)$ ($i_{dR}(G)$) for a given graph G. Double Roman Domination was introduced by Robert Beeler, Tereza Haynes and Stephen Hedetniemi in 2016 as a stronger version of a previous studied problem, called Roman domination, and this concept is motivated by a historical problem on military defense. Double Roman domination was studied for a few families of graphs with $\Delta(G) = 3$, such as the generalized Petersen graphs. However, there are many classes of cubic graphs for which this parameter has not been investigated yet, such as the snarks. In this work, we prove that the decision version of the double Roman domination problem is $\mathcal{NP}$-complete even when restricted to planar bipartite graphs with maximum degree 3. Given this result, we investigate $\gamma_{dR}(G)$ and $i_{dR}(G)$ for families of graphs with maximum degree 3. We determine the (independent) double Roman dominating number for the families of Flower Snarks and Generalized Blanuša Snarks and we present tight lower and upper bounds for the (independent) double Roman dominating number of Goldberg snarks and Loupekine Snarks.

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Independent locating-dominating sets in some graph classes

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Keywords: independent set, dominating set, locating-dominating set.

Consider the situation where a graph G models a facility or a multiprocessor network with limited-range detection devices placed at chosen vertices of G. The purpose of these devices is to detect and precisely identify the location of an undesirable encroachment such as a thief, saboteur, fire, or faulty processor that may suddenly be present at any vertex. The aim is to determine the vertex locations of the minimum number of devices to locate an intruder at any vertex precisely.

Formally, a vertex subset S is termed *locating-dominating* if it is dominating and no two distinct vertices of $V(G) \setminus S$ have the same open neighbourhood in S ($N_G(u) \cap S \neq N_G(v) \cap S$, $\forall\ u, v \in V(G) \setminus S$, $u \neq v$). $V(G)$ is a locating-dominating set for any graph G. S is termed *independent locating-dominating* (ILD-set) if it is independent and locating-dominating. The cardinality of the smallest independent locating-dominating set is denoted by $ILD(G)$. The problem was first introduced by Slater and Sewell [2018], who reported results for 3-regular infinite ladders and trees and also proved that simply deciding if a graph G has an ILD set is an NP-complete problem.

We investigate independent locating-dominating sets in some graph classes such as split, cycles-related graphs, maximal planar graphs, and bipartite graphs. We present closed formulas for $ILD(G)$ and complexity results for these graphs.

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Session 13

Jogos partizan de convexidade em grafos

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Keywords: Convexidade de grafos, Jogos geodésicos, PSPACE-completude, Teoria dos jogos combinatórios

O primeiro artigo de convexidade em grafos gerais, em inglês, é o artigo "*Convexity in graphs*", publicado em 1981. Um de seus autores, Frank Harary, introduziu em 1984 os primeiros jogos de convexidade em grafos, focados na convexidade geodésica, que são jogos imparciais e foram investigados em uma sequência de cinco artigos até 2003. Após 20 anos de inatividade nessa linha de pesquisa, os autores deste artigo provaram o primeiro resultado de complexidade PSPACE em jogos imparciais de convexidade [1], bem como obtiveram algoritmos polinomiais para árvores usando a Teoria de Sprague-Grundy dos anos 1930. Neste artigo, apresentamos as variantes partizan desses jogos imparciais, inicialmente na convexidade geodésica e depois estendidas para outras convexidades de grafos, obtendo estratégias vencedoras e resultados de complexidade PSPACE. Entre elas, obtemos estratégias vencedoras para geometrias convexas gerais e estratégias vencedoras para árvores a partir da Teoria Combinatória de Jogos, proposta por Conway nos anos 1980, fortemente baseada no conceito dos números surreais. Provamos também que a variante normal e a variante misère do jogo partizan da envoltória convexa são PSPACE-difíceis na convexidade geodésica mesmo em grafos com diâmetro dois.

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Convexity Games on Oriented Graphs

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Keywords: Convexity in Graphs, Oriented graphs, Convexity Games

In 1984, Harary introduced the concept of graph convexity games focused on the geodesic convexity. In 2024, Araújo et al. [1] proved the first PSPACE-hardness results on impartial convexity games and expanded the study to include different graph convexities. In this paper, we expand the scope of this research by incorporating oriented graphs. We explore the complexities of determining the winner of convexity games played in oriented graphs. First, we introduce concepts pertinent to convexity games in oriented graphs, including the interval function, convex hull, and various graph convexities such as geodesic, P_3, and monophonic. Then we show that deciding the winner of the normal and misère variants of the hull game and closed hull game in the P_3-convexity is PSPACE-complete. Specifically, we provide a reduction from the Clique Forming game, a known PSPACE-complete problem. We construct an oriented graph from a given instance of the Clique Forming game in such a way that the winner of the convexity game corresponds to the winner of the Clique Forming game instance. Additionally, we discuss polynomial time solvability under certain conditions. For example, in transitive tournaments, we establish that Alice wins the closed hull game in the P_3-convexity if and only if the number of vertices of the tournament is odd. Intuitively, this observation arises from the fact that an odd number of vertices allows Alice to partition them into two equal-sized sets with her initial move, thereby securing victory by using the *mirror strategy.* Conversely, when the vertex count is even, Bob can respond maintaining balance between the two sets.

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Results on f-reversible processes[1]

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Keywords: Reversible Process, Infection, Critical Set, Conversion Set

Given a graph G and a function $f : V(G) \to \mathbb{N}$, we study a type of iterative process on G, introduced in (Dourado et al. Reversible iterative graph processes. Theor. Comput. Sci., **460**, 16–25, 2012) with the name of f-reversible process which can model the spread of an infection among a population and consists in, given an initial vertex labeling $c_0 : V(G) \to \{0, 1\}$, each vertex v changes its label at time $t = 1, 2, 3, \ldots$ if and only if v has at least $f(v)$ neighbors with the opposite label at time $t - 1$. We denote such processes by the sequence $(c_t)_{t\in\mathbb{N}} = c_0, c_1, c_2, \ldots$, such that $c_t : V(G) \to \{0, 1\}$, where $c_t(v)$ denotes the ***state*** of v, at time t. We say that an f-reversible process $(c_t)_{t\in\mathbb{N}}$ converges at time t if t is the least natural such that, for every $t' \geq t$, $c_{t'}(v) = 1$ for every vertex v. Besides this version on graphs, we also work with digraphs, where a vertex takes into account only its inner neighbors to define its state in the next time step. We study the problems of determining the minimum cardinality $r_f(G)$ of an ***f-conversion set*** *of a digraph D*, which is a vertex subset S such that, if an f-reversible process $(c_t)_{t\in\mathbb{N}}$ is such that $c_0(v) = 1$ if and only if $v \in S$, then the process converges at some finite time, and of determining the minimum cardinality $r^c_f(G)$ of a special type of f-conversion set that we called ***f-critical set*** *of a graph G*, which is an f-conversion set S of G such that, if an f-reversible process $(c_t)_{t\in\mathbb{N}}$ is such that $c_0(v) = 1$ if and only if $v \in S$, then the process converges at time at most 1. When Dourado et al. introduced the concept, they also proved, among other results, that determining whether $r_f(G) \leq k$ when the thresholds are all equal to 2 is NP-hard, but polynomial for certain types of paths and cycles. We prove that $r^c_f(G)$ can be computed in linear time when G is a path and, in the directed version, we show that determining whether $r_f(D) \leq k$ is NP-hard even if all thresholds are equal to 1, there exists only one directed cycle in D, and D is an orientation of a subcubic graph.

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W-Hierarchy of Geodetic number problem[‡]

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Keywords: Geodetic number, W-hierarchy, bipartite graphs.

Abstract

The *interval* $I[u, v]$ between two vertices u and v in G is the set of vertices of G that belong to a shortest path between u and v. For a set S of vertices, the interval $I[S]$ of S is the union of the intervals $I[u, v]$ over all pairs of vertices u and v in S. A set of vertices S is considered *geodetic* if $I[S]$ contains all the vertices of G. The *geodetic number* $g(G)$ of a graph G is defined as the minimum cardinality of a geodetic set. The problem of determining the geodetic number of a graph is an NP-hard problem with its recognition problem stated as an NP-complete problem, as established by Harary et al. (1981) and Dourado et al. (2010). While classic complexity theory aims to identify problems solvable in polynomial time, practical scenarios often require approximate algorithms for intractable problems. In 1999, Downey et al. introduced the Parameterized Complexity, which investigates the existence of algorithms whose exponential complexity depends only on certain specific aspects of the input. They also developed classes of parameterized problems, organized in the W hierarchy, In this work, we show that the GEODETIC NUMBER problem belongs to the complexity class $W[2]$ through a reduction to WEIGHTED CIRCUIT SATISFIABILITY WITH WEFT t AND DEPTH h (WCS(t, h)). Moreover, we show that the GEODETIC NUMBER problem is $W[2]$-hard for bipartite graphs, achieved through a reduction from the DOMINATING SET problem.

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Session 14

Almost All Complementary Prisms Have Many Pairwise-disjoint Perfect Matchings

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Keywords: Graph classes, Graph colouring, Matching, Graph factorisation

Being G a simple graph, the complementary prism $G\overline{G}$ is the graph obtained by taking a copy of G and a copy of its complement $\overline{G}$, and then linking with an edge each pair of corresponding vertices. ZCGZMF (Zatesko, Carmo, Guedes, Zorzi, Machado, and Figueiredo, *On The Chromatic Index of Complementary Prisms*, EUROCOMB 2019) showed that every non-regular complementary prism $G\overline{G}$ is *Class 1* (i.e. $\Delta(G\overline{G})$-edge-colourable) and conjectured that the Petersen graph is the only *Class 2* (i.e. not *Class 1*) complementary prism (we use $\Delta(G)$ for the maximum degree of graph G and $d(G)$ for the degree of G when G is regular). We provide further evidence for this conjecture by showing that asymptotically almost every d-regular complementary prism has at least $4\lfloor h/3 \rfloor + (h \bmod 3)$ pairwise-disjoint perfect matchings (PDPMs), being $h = (d-3)/2$. Determining the maximum number of PDPMs that a d-regular graph G has is NP-hard, since it generalises the problem of deciding if G is *Class 1* (i.e. has d PDPMs). Actually, even deciding if G has 2 PDPMs has been shown to be NP-complete (Zatesko, Carmo, Guedes, Machado, and Figueiredo, *The Hardness of Recognising Poorly Matchable Graphs and the Hunting of the d-Snark*, to appear in RAIRO Operations Research). A key ingredient in the proof of our result is the fact that a.a.e. regular n-vertex graph G is hamiltonian when $3 \le d(G) \le n-4$ (Krivelevich, Sudakov, Vu, and Wormald *Random Regular Graphs of High Degree*, Random Struct. Alg. 2001). Since, as shown by ZCGZMF, a d-regular complementary prism $G\overline{G}$ on $2n$ vertices has $n \equiv 1 \pmod 4$ and $d(G) = d(\overline{G}) = (n-1)/2 = d-1 \equiv 0 \pmod 2$, we have that a.a.s. both G and $\overline{G}$ have $h = (d-3)/2$ pairwise-edge-disjoint hamiltonian cycles each, from which we show how to obtain h PDPMs in $G\overline{G}$. We improve this amount by finding a way of obtaining 4 PDPMs for each set consisting of 3 of these cycles in G and 3 of these cycles in $\overline{G}$, yielding the result.

Extending a perfect matching to a hamiltonian cycle in some classes of graphs

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Keywords: perfect matching, hamiltonian cycle, hypercube, product of graphs

Let $G = (V(G), E(G))$ be a simple, connected and undirected graph of even order. Let K_n be a complete graph with n vertices. A *matching* M in G is a set of pairwise nonadjacent edges, and it is called *perfect* when every vertex of G is incident with some edge of M. We say that M *extends to a perfect matching* if there exists $N \subseteq E(G)$ such that $M \cup N$ is a perfect matching of G. Similarly, M *extends to a hamiltonian cycle* if there exists $N \subseteq E(G)$ such that $M \cup N$ induces a hamiltonian cycle of G.

In 1993, Ruskey and Savage conjectured that every matching in a hypercube Q_d, $d \geq 2$, can be extended to a hamiltonian cycle. In 2007, Fink proved a restricted version, known as Kreweras's Conjecture, in which M is a perfect matching. In fact, Fink showed a stronger theorem: for every perfect matching M of $K_{|V(Q_d)|}$, there is a perfect matching N in Q_d such that $M \cup N$ extends to a hamiltonian cycle of $K_{|V(Q_d)|}$. That is, every perfect matching of $K_{|V(Q_d)|}$ can be extended to a hamiltonian cycle using only edges of Q_d. This result opened up a new avenue of investigation and new problems emerged. One of them is to expand Fink's idea to other classes of graphs, by formalizing the following property. A graph G is *Pairing-Hamiltonian (PH)* if, for every perfect matching M of $K_{|V(G)|}$, there is a perfect matching N of G such that $M \cup N$ induces a hamiltonian cycle of $K_{|V(G)|}$.

This property has been studied for some classes of graphs, in particular for cartesian products of graphs. In 2015, Alahmadi et al. characterised in which cases $K_m \square P_n$ and $K_m \square C_n$ are PH. Furthermore, the authors showed that $P_n \square Q_d$ and $C_n \square Q_d$ are PH for every $n \geq 1$ and $d \geq 5$. However, for $d \in \{3, 4\}$, the problem is still open. In this work, we show that, if $P_n \square Q_3$ is PH, then $P_n \square Q_4$ is also PH. Moreover, we also proved that $P_3 \square Q_3$ and $C_4 \square Q_3$ are PH, and that if G_1 and G_2 are PH, then their join, $G_1 \vee G_2$, is also PH.

The cost of perfection for matchings in Prism graphs

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Keywords: Matching, perfect matching, cubic graph

This topic was inspired by the application of triangular meshes in the modeling of solid objects in computer graphics. The focus was on converting these meshes into quadrangulations by joining adjacent triangles, approached as a graph matching problem. Given a graph $G = (V, E)$ that admits a perfect matching, we consider the minimum of the ratio between the maximum weight of a perfect matching and the maximum weight of a general matching over all possible edge weight functions. The parameter is defined as $\eta(G) = \min_{w:E\to R^+} \frac{w(P^*(G))}{w(M^*(G))}$, in which $w : E \to R^+$ be the weight of the edges of G. Given a subset $E' \subseteq E$, we refer to the quantity $w(E') = \sum_{e\in E'} w(e)$ as weight of E'. A maximum weight matching is a matching $M^*(G)$ of maximum possible weight in G. A maximum weight perfect matching is a perfect matching $P^*(G)$ of maximum possible weight, among all perfect matchings of G.The parameter η was introduced in [1], and due to this application, the first study focused on cubic graphs. We investigate the Prism graphs, generated by the cartesian product between C_n (a cycle of n vertices, where $n \geq 3$) and the complete graph K_2. These graphs are cubic and have perfect matchings. We prove that for Prism graphs with an even number of vertices, $\eta = 2/3$, and for Prism graphs with an odd number of vertices, $\eta = 1/2$.

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On the ℓ-Nested Cut problem: a generalization of Matching Cut

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Keywords: matchings; edge cuts; parameterized complexity

In the MATCHING CUT problem, we are given a finite simple graph G and must decide if there exists a matching $M \subseteq E(G)$ of G such that $G \setminus M$ is disconnected. Chvátal [1] showed that MATCHING CUT is NP-complete for graphs with maximum degree at least 4 and presented a polynomial time algorithm for subcubic graphs. MATCHING CUT has attracted considerable attention lately, particularly in the parameterized complexity community [2].

In this work, we investigate a novel generalization of MATCHING CUT, which we have called the ℓ-NESTED CUT problem. Now, we are also given an integer ℓ and want to decide if there is a matching $M \subseteq E(G)$ such that $G \setminus M$ has at least ℓ connected components. We begin our study by showing a polynomial time algorithm for chordal graphs and a linear time algorithm for split graphs. On the other hand, for unipolar graphs, a superclass of split graphs, we prove that the problem is W[1]-hard when parameterized by ℓ and the number of edges in the cut. Finally, we show that, unlike in MATCHING CUT, ℓ-NESTED CUT remains NP-hard on subcubic graphs. Our current efforts point to an FPT algorithm for subcubic graphs when parameterized by ℓ and some positive results on enumerative kernelization for the problem.

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Session 15

Parameterized algorithms for thinness via the cluster module number

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Keywords: Thinness – Vertex cover – Cluster module number – Parameterized complexity – Graph parameters – Polynomial kernels.

In 2007, Mannino et al. defined k-thin graphs as a generalization of interval graphs, and defined the thinness of a graph to be the minimum k such that the graph is k-thin. When given a k-thin representation of a graph, several NP-complete problems can be solved in XP time parameterized by k. Thus, the problem of computing the thinness of a graph, as well as the corresponding representation, has various algorithmic applications. In this work we define a new graph parameter that we call the *cluster module number* of a graph, which generalizes twin-cover and neighborhood diversity, and can be computed in linear time. We then present a linear kernel for the problem of calculating the thinness on graphs with bounded cluster module number. As a corollary, this results in a linear kernel for Thinness when the input graph has bounded neighborhood diversity, and exponential kernels when the input graph has bounded twin-cover or vertex cover. On the negative side, we observe that Thinness parameterized by treewidth, pathwidth, bandwidth, (linear) mim-width, clique-width, modular-width, or the thinness itself, has no polynomial kernel assuming $\mathsf{NP} \not\subseteq \mathsf{coNP/poly}$.

Chain Traveling Salesmen Problem

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Keywords: Cooperative TSP, Parameterized Algorithms, Complexity, Graph Exploration

Motivated by swarm robotics, we introduce a variant of the Traveling Salesmen Problem (TSP), which we call the *Chain-TSP*. We are given a connected graph G on n nodes and want to route a set of k agents so that every vertex is visited at least once by any of the agents. At each time step $j = 0, 1, \ldots,$ agent $i = 1, \ldots, k$ is located at some vertex p_i^j, and can either keep the current position or move to a neighbor vertex. Also, at any moment, the sequence of agents' positions must form a *chain*, that is, for each $1 \leq i \leq k-1$, either $p_i^j = p_{i+1}^j$ or p_i^j and p_{i+1}^j are adjacent in G. The goal is, given a chain p^0 corresponding to the agents' initial positions, to find a sequence of chains $p^1, p^2, \ldots, p^t$ that visits all the vertices of G and minimizes the total time t. In the decision version, one is also given a time limit T and is asked whether there is a solution that takes time at most T.

This problem is NP-hard already for $k = 1$ as there is a solution of total time $T = n-1$ if and only if there is a Hamiltonian path in G starting at p_1^0. Additionally, for $k = 2n$, we show that it is hard to decide whether a solution with one step exists or if at least two are needed. The proof uses a simple reduction from the Hamiltonian path problem. Therefore, it is NP-hard to obtain an approximation factor better than 2 for Chain-TSP if $k = O(n)$.

From the parameterized point of view, using the number of agents k or the time limit T as a parameter does not make the problem any easier because the the problem remains NP-hard when they are bounded by a constant. Combining both parameters, however, makes the problem trivially Fixed-Parameter Tractable (FPT), as there is no solution if $n > Tk$, and we can use brute force if $n \leq Tk$. By considering structural parameters of the graph, we get a more interesting result:

Theorem 1. *The Chain-TSP problem, without vertex repetition by individual agents, is FPT parameterized by the treewidth of the input graph plus k.*

Session 16

On open-independent dominating sets for some subclasses of cubic graphs

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Keywords: *open-independent dominating set, cubic graphs*

Let G be a finite, simple, and undirected graph and let S be a set of vertices of G. The *open neighborhood* of vertex v is $N(v) = \{w \in V(G); vw \in E(G)\}$ and the *closed neighborhood* $N[v] = N(v) \cup \{v\}$. The vertex set S is *dominating* if $\bigcup_{v \in S} N[v] = V(G)$.

In some cases the subgraph induced by a dominating set S is required to have an additional property such as independence, paired or connected. The set S is *independent* if no two vertices in S are adjacent, that is, for every $v \in S$, $|N[v] \cap S| \leq 1$. We define S to be *open-independent* if for every $v \in S$, $|N(v) \cap S| \leq 1$. An open-independent dominating set (OIND-set) is a set that is both an open-independent set and a dominating set, then, a set $S \subseteq V(G)$ is an open-independent dominating set if S is a dominating set for which each component of the subgraph induced by S has cardinality at most two. We let $\gamma_{oind}(G)$ denote the minimum cardinality of an open-independent dominating set of G.

OIND sets were introduced by (1) and, in fact, few results have been established. In this work we study the γ_{oind} for some subclasses of cubic graphs. More specifically, we show that if G is a circular ladder graph then $\gamma_{oind} = \lceil \frac{|V(G)|}{4} \rceil + 1$ if $|V(G)| \equiv 4 \mod 8$ and $\gamma_{oind} = \lceil \frac{|V(G)|}{4} \rceil$ otherwise. And for möbius ladder graph we prove that $\gamma_{oind} = \lceil \frac{|V(G)|}{4} \rceil + 1$ if $|V(G)| \equiv 0 \mod 8$ and $\gamma_{oind} = \lceil \frac{|V(G)|}{4} \rceil$ otherwise.

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Applying Spectral Graph Theory to Coupled Oscillation problems

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Keywords: Spectral Graph Theory, Laplacian matrix, Eigenvalues, Eigenvectors, Computational physics, Coupled oscillations

In this paper, we show a novel approach to solving coupled oscillation problems using Spectral Graph Theory. To this end, we show how to model various types of coupled oscillators using graphs and how to obtain the natural frequencies and the normal modes of oscillation from the eigenvalues and eigenvectors of the Laplacian matrix of the considered graphs. From this, we obtain for the first time the natural frequencies and the normal modes of various spring mass systems for any number n of blocks, such as straight line, circular and with or without walls. This approach also helps to obtain results in Spectral Graph Theory for certain graph classes from the normal modes of the corresponding coupled oscillators of the graphs. We also obtain computer simulations showing the normal modes of several coupled oscillators.